THE FLAVOUR SEQUENCE AND SUPERSELECTION RULES

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In the lepton (flavour) sequence $e$, $\mu$, $\tau$, ... the extra mass of the muon and of the other heavy leptons may be interpreted as coming from a non-zero vacuum expectation value of the electromagnetic field $\phi$ (similar to the mass derivation by Higgs mechanism from a vacuum expectation value of a scalar field). In this way a simple derivation of a generalized renormalizable QED $[2]$ for the electron-muon system is obtained. In ref. [2] the two subspaces of states describing the electron and muon, respectively, are separated by a superselection rule in such a way that only pairs from one of the two sectors, e.g. ($\psi^- \psi^+$), can be connected to the other one by electromagnetic process, or vice versa. The energy is definite positive and in each sector the ordinary QED holds. A single fermionic field $\psi$ is introduced and the theory accounts for the equal charges of the fermions. Since the muon and the other heavy leptons are viewed as different states (excitations) of the field $\psi$, they should already be contained in QED and the renormalization procedure should be formulated in such a way as to incorporate also the muon sector, the $\tau$ sector, etc. We will see later that the present procedure to obtain this generalized QED can be useful in the formulation of such a new renormalization procedure.

We further discuss the indefinite metric problem and show that it can be dealt with by giving a structure to the vacuum of the theory; from this a superselection rule will emerge in a natural way.

We start with the standard electromagnetic theory

$$\{-i\gamma^\mu \partial_\mu - m - e\gamma^\mu A_\mu (x)\} \psi (x) = 0; \Box A_\mu (x) = e\gamma_\mu \psi$$

(1)

where $m$ is the bare mass of the electron, the lowest state of leptonic matter. When the theory is renormalized it should contain also other leptons besides the physical electron and the physical photon. For this purpose, we assume that
With \( \langle A \rangle \) such that
\[
\mathcal{L}(\partial_{\mu} - e A_{\mu}^\nu) C \psi = e \langle A \rangle \psi
\]
where \( C = C(\gamma^\mu \tilde{D}_{\mu}^\nu), \tilde{D}_{\mu}^\nu = (\partial_{\mu} - e A_{\mu}^\nu) \). Usually \( C \) is assumed to be zero. Here we must determine it. In order that the \( \psi \) field admits an excitation with mass \( k \pm m \), it must also satisfy the equation
\[
(-i \gamma^\mu \tilde{D}_{\mu}^\nu - k) \psi = 0
\]
Operating with \( C \) on (4) we have
\[
-\mathcal{C} \gamma^\mu \tilde{D}_{\mu}^\nu \psi = k C \psi
\]
By inserting (2) and (3) into (1) we have also
\[
(-i \gamma^\mu \tilde{D}_{\mu}^\nu - m) \psi = i \gamma^\mu \tilde{D}_{\mu}^\nu C \psi
\]
Thus, from (5) and (6),
\[
C \psi = -\frac{1}{k} \mathcal{C} \gamma^\mu \tilde{D}_{\mu}^\nu \psi = -\frac{1}{k} (-i \gamma^\mu \tilde{D}_{\mu}^\nu - m) \psi
\]
where the commutator of \( C \) with \( \gamma^\mu \tilde{D}_{\mu}^\nu \) has been dropped since it gives a Pauli term which does not affect the mass spectrum \([2]\). By using (7), it is now possible to write (6) as
\[
(-i \gamma^\mu \tilde{D}_{\mu}^\nu - m) (-i \gamma^\mu \tilde{D}_{\mu}^\nu - k) \psi = 0
\]
which is the generalized Dirac equation proposed in the generalized renormalizable QED to study the electron-muon system \([2]\). The procedure can be continued successively to the excitation of the muon and so on.

It is well known that the "higher order" equation (8) leads to the difficulties of indefinite metric and of physical interpretation. To face such problems we introduce \([3]\) the physical vacuum \( |0_e> \) for the electron sector as
\[
|0_e> = \prod_{\sigma} ( |\mu^- e_{\mu} \sigma \rangle + |\mu^+ e_{\mu} \sigma \rangle )^\dagger |0> \]