1. Introduction

The decision tree model is one of the most powerful models for obtaining the lower bound time complexity of a given computational (or decision) problem. In this model the computation process involves selecting a path down a (ternary) tree, depending on the outcome of certain test functions applied along the way. When a leaf is reached the algorithm must be able to give an answer.

The methods commonly used in purpose to derive the lower bounds may be divided on two major groups; (A) ones based on the information-theoretic bounds of a given problem, (B) ones based upon the specific properties of the tests performed in the decision tree algorithm. In the latter the tests are mostly restricted to the class of linear polynomials, except the elegant paper of Yao [10] where the polynomials of degree 2 were allowed. The object of the present paper is to offer a new uniform method which enables us to deal with the wider class of the decision trees. Namely, the only restriction we put on the model is that the allowed tests must be polynomials with a bounded number of the irreducible factors (but they can be of any degree).

Given computational (or decision) task we consider a corresponding array of the polynomial inequalities called the (constructive) description. From the particular properties regarding a size and a so-called $\mathbb{M}$-redundancy of this array the lower bound of the problem can be inferred. Thus, the proposed method consists of studying the algorithm cost by reducing this to the combination of combinatorial, analytic and algebraic problems.
It is worth mentioning that our research was inspired by the fundamental paper of Rabin [6].

In Section 2 some needed definitions are presented. The main theorem is formulated in the Section 3. An application of the offered methods is given in Section 4.

2. Preliminaries and Definitions

In the sequel we will need some rudimentary knowledge of algebraic geometry. In order to make the paper understandable some elementary definitions are presented in this section.

Let \( \mathbb{R} \) be a field of reals and \( \mathbb{R}[x_1, x_2, \ldots, x_d] \) be a ring of polynomials in \( d \) indeterminants \( x_1, x_2, \ldots, x_d \) with coefficients in \( \mathbb{R} \). Any set \( \{ p_i \} \) of polynomials defines a real variety \( V = V(\{ p_i \}) = \{ x \in \mathbb{R}^d : each \ p_i(x) = 0 \} \).

A variety \( V \) is irreducible if \( V = V_1 \cup V_2 \) (\( V_1, V_2 \) varieties) implies \( V = V_1 \) or \( V = V_2 \). The dimension \( \dim V \) of a variety \( V \) is the maximal integer \( n \) such that there exist distinct irreducible varieties \( V_0, V_1, \ldots, V_n \) such that \( V_0 \subset V_1 \subset \ldots \subset V_n \subset V \).

In the special case when \( V = V(p) \) is a set of zeros of one polynomial \( p \) and \( \dim V = d-1 \) the variety \( V \) is called hypersurface. If additionally \( p \) is a linear polynomial \( V \) is called hyperplane.

Let us introduce the following definition which is necessary because \( \mathbb{R} \) is not algebraically closed:

We say that an irreducible hypersurface \( V \subset \mathbb{R}^d \) has the identity property if for any irreducible variety \( V_1 \) and any open set \( U \subset \mathbb{R}^d \) the equality \( V \cap U = V_1 \cap U \neq \emptyset \) implies \( V = V_1 \).

All irreducible varieties \( V \subset \mathbb{C}^d \), \( \mathbb{C} \) the field of complex, have the identity property. Remark also that all hyperplanes have the identity property. Further facts considering algebraic geometry one can found e.g. in Kendig [5], van der Waerden [9].

Throughout the paper all functions denoted by \( p(x), d(x) \) with indices are polynomials in \( \mathbb{R}[x_1, x_2, \ldots, x_d] \).

A system \( \mathbf{IN}_i \) of inequalities \( d_{i1}(x) \geq 0, \ldots, d_{ik}(x) \geq 0 \) is called simultaneously non-negative for \( x \in D \subset \mathbb{R}^d \) (shortly \( SP(\mathbf{IN}_i, x) \)) if the conjunction \( d_{i1}(x) \geq 0 \& \ldots \& d_{ik}(x) \geq 0 \) is true.

Given an \( m \times k \) array \( \mathbf{IN} \) of inequalities