Since the development of quantum mechanics (Q.M.), there have been repeated suggestions that Q.M. could be completed by an underlying substructure, as classical thermodynamics is underlied by statistical mechanics. One knows for instance the hidden variable theories of D. Bohm, or of L. de Broglie, tending to restore determinism. The place of Einstein is different in this context. The famous E.P.R. paper [1] starts with a realistic point of view and that there are no action-at-a-distance. Then, considering a special situation (see lower), the authors demonstrate that Q.M. is not complete. BOHR's reply [2] rejects the realistic point of view. We will find similar arguments (realistic picture and locality) in the demonstration of BELL's theorem.

I. BELL's theorem.
I.1. Motivations for the formalism.

Let us consider the BOHM'S version of "E.P.R. Gedankenexperiment" (Fig. 1).

Two particles with spin 1/2 are produced in a singlet state (null total spin) and separate. Two Stern-Gerlach filters allow to measure the spins components $S_α$ and $S_β$ of the two particles along directions $α$ and $β$, yielding results +1 or -1 (in $\hbar/2$ units). Elementary Q.M. calculations yield predictions on various measured quantities such as $P_+(α)$ (probability of finding 1 in channel + of apparatus I in orientation $α$) or $P_-(α, β)$ (probability of joint detections in channel + and - of apparatuses I and II in orientations $α$ and $β$).

In the special case $(α, β) = 0$ one finds $P_+ = P_- = 1/2$, while $P_+(α) = 1/2$. We are thus led to the conclusion that particle 1 has 50 % chances of being found in channel +, but if so, then the conditional probability of finding 2 in...
channel - is 100 %. This is a rather strong correlation between two distant measurements. If we ask "How does it work?", Q.M. does not afford any illuminating answer (as emphasized by BOHR, it is not its purpose!). But we can try an explanation that would also hold in classical physics. The two particles of one pair are supposed to bring a common property, shared during the preparation in S. If we suppose that this property determines the results in I and II, we have a clear picture for "explaining" the correlations. We can hope to recover Q.M. by averaging over the ensemble of emitted pairs. At this level, the value of such a (classical) picture is only a matter of taste.

I.2. BELL's Theorem [3]. Let us denote by \( \lambda \) the common property of the two particles of a pair (\( \lambda \) is not restricted to be a scalar). Following our picture, the results of measurements on this pair depend on \( \lambda \), and can be written

\[
S_\alpha = A(\lambda, \alpha) = \begin{cases} 
+1 & \text{for I (orientation } \alpha) \\
-1 & \text{for II (orientation } \beta) 
\end{cases}
\]

\[
S_\beta = B(\lambda, \beta) = \begin{cases} 
+1 & \text{for I (orientation } \alpha) \\
-1 & \text{for II (orientation } \beta) 
\end{cases}
\]

We then describe the ensemble of pairs emitted with a density of probability \( p(\lambda) \) such that

\[
p(\lambda) \geq 0 \text{ and } \int d\lambda p(\lambda) = 1
\]

Thanks to this formalism, the various results of the possible experiments can be expressed. For instance,

\[
P_+ (\alpha) = \int d\lambda p(\lambda) \left[ \frac{1}{2} A(\lambda, \alpha) + 1 \right]
\]

An interesting quantity (for exhibiting the correlations) is the expectation value of the product \( S_\alpha S_\beta \)

\[
S_\alpha S_\beta = E(\alpha, \beta) = \int d\lambda p(\lambda) A(\lambda, \alpha) B(\lambda, \beta)
\]

which is also equal to

\[
E(\alpha, \beta) = \left[ P_{++}(\alpha, \beta) + P_{--}(\alpha, \beta) \right] - \left[ P_{+-}(\alpha, \beta) + P_{-+}(\alpha, \beta) \right]
\]

Starting from (1), (2) and (3) elementary algebra leads to

\[
-2 \leq s \leq 2 \quad \text{where}
\]

\[
S = E(\alpha, \beta) - E(\alpha', \beta) + E(\alpha, \beta') + E(\alpha', \beta')
\]

(\( S \) involves 4 different situations). (4) is the Bell inequality obtained by