REMARKS ABOUT PERFORMANCE PROFILES

by

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1. Introduction

The term "performance profile" was first assigned its present technical meaning in the field of numerical software in 1976. It was introduced and studied in an attempt to understand some of the results obtained (Kahaner 1971) when evaluating numerical quadrature routines. Since then it has been helpful in designing techniques for evaluating optimization routines (Lyness and Greenwell 1977, Lyness 1979) and it appears that analogous techniques may be applied to evaluate other items of numerical software. It also provides insight to the behavior of programs in which results from one routine are used as input to another routine. This has been termed the "interface problem."

In section 2 of this paper, performance profiles are described in general, and in section 3, an example of a jagged performance profile is given. In sections 4 and 5, their relevance to techniques for software evaluation and to an interface problem is briefly described.

This paper is intended to introduce and illustrate some of the basic ideas in this area. For a detailed description of some of the applications of these ideas, the reader should examine some of the references listed at the end of the paper.

2. The Performance Profile

In concept, the term performance profile is extremely simple.

One requires a one parameter (usually denoted by \( \lambda \)) set of numerical problems, called a problem family for which the exact solution of each problem depends continuously on \( \lambda \). One requires an item of numerical software, allegedly capable of providing a numerical approximation to the solution of each member of this problem

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family. A performance profile is a plot of some aspect of the performance of this item of numerical software, the abscissa being \( \lambda \).

These guidelines are very broad. At present only a narrow subclass of performance profiles have been used by the author to help to understand the behavior of numerical software.

To construct a performance profile we need to state unambiguously a problem family (which depends on \( \lambda \)), a software item which can handle these problems, and the aspect of the performance in which we are interested. I shall now describe a particular choice for these three items.

The problem family comprises a set of quadratures

\[
\int_{\lambda}^{2} f(x, \lambda) \, dx \quad 1 \leq \lambda \leq 2
\]

where

\[
f(x, \lambda) = 0.1/((x-\lambda)^2 + 0.01).
\]

Note that the true solution

\[
\int_{\lambda}^{2} f(x, \lambda) \, dx = \arctan(10(2-\lambda)) - \arctan(10(1-\lambda))
\]

is an analytic function of \( \lambda \), and so depends continuously on \( \lambda \) as required by the guidelines for a performance profile.

The item of numerical software is a simple routine which returns the \( N \) point Gauss-Legendre quadrature rule result, used with \( N=10 \), i.e.

\[
Q_{\lambda} f(x, \lambda) = \sum_{i=1}^{10} \omega_{i} f(x_{i}).
\]

The aspect of the performance in which we are interested is the (unsigned) error made by applying this software, i.e.

\[
E(\lambda) = Q_{\lambda} f(x, \lambda) - \int_{\lambda}^{2} f(x, \lambda) \, dx.
\]

Since the exact integral can be expressed in terms of standard functions, there would be little difficulty involved in writing a short program to evaluate \( E(\lambda) \) for values of \( \lambda \) between 1 and 2 and then plotting \( E(\lambda) \) as a function of \( \lambda \). This plot is a performance profile and is illustrated in figure 2. It is clearly a continuous