1. Introduction

In 1956 Utiyama proposed to introduce the gravitational field as the gauge field of the Lorentz group.\(^1\) He introduced 24 fields by generalizing 6 constant parameters $\omega_{ij} (= - \omega_{ji})$ for homogeneous Lorentz transformations to arbitrary functions $\omega_{ij}(x)$. Later Kibble considered all the 10 parameters of the inhomogeneous Lorentz group (Poincaré group), and laid the basis for Poincaré gauge theory with 40 independent field variables.\(^2\)

Hayashi and Nakano proposed to extend translations only, leaving the six parameters $\omega_{ij}$ constant.\(^3\) In this translation gauge theory, 16 field variables $c_k^\mu$ are introduced as the gauge field, by requiring that the action integral be invariant under the group of extended translations and global Lorentz transformations. This invariance group is the simplest one that includes the group of general coordinate transformations.

The underlying space-time of the translation gauge theory is the Weitzenböck space-time with absolute parallelism. The notion of absolute parallelism was first introduced into physics by Einstein, trying to unify gravitation and electromagnetism.\(^4\) His attempt failed because there was no Schwarzschild solution in his field equation.\(^5\) A purely gravitational theory based on the Weitzenböck space-time was revived by Möller,\(^6\) and its Lagrangian formulation was given by Pellegrini and Plebanski.\(^7\)

The theory of gravitation based on the Weitzenböck space-time was extensively studied by Hayashi and Shirafuji,\(^8\) and it was given the name, new general relativity, since Einstein in 1928, after inventing general relativity, considered absolute parallelism for the first time, and since its main consequences were analogous to those of general relativity so far as macroscopic phenomena were concerned.

2. Fundamental particles and translation gauge group

We start from the action integral in special relativity for the fundamental particles of spin $1/2$, \(\psi_{\mu}^a(x)\), and the relativistic wave equation is written as

\[
\Box \psi_{\mu}^a(x) = i \gamma_\mu \frac{\partial \psi_{\mu}^a(x)}{\partial x^\mu} - m \psi_{\mu}^a(x) = 0
\]
\[ S_M = \int d^4x \ L_M(q, \partial_k q) , \] (2.1)

which is invariant under Lorentz transformations,

\[ \delta x^k = c^k + \omega^k_j x^j , \quad (\omega_{kj} = - \omega_{jk}) , \] (2.2a)

\[ \delta q = (i/2) \omega_{ij} S^{ij} q , \quad \delta (\partial_k q) = (i/2) \omega_{ij} S^{ij} (\partial_k q) + \omega^k_j (\partial_j q) , \] (2.2b)

where \( S^{ij} \) are the Lorentz generators, and \( c^k \) and \( \omega_{ij} \) are independent, constant 10 parameters. Here \( q \) collectively denotes quarks and leptons, and the Minkowski metric \( \eta_{ij} \) is given by diag(-1,+1,+1,+1).

We now extend translations to extended translations (namely, to general coordinate transformations) for which the parameters \( c^k \) are arbitrary functions of space-time points, and demand that the action integral should be invariant under general coordinate transformations and under global Lorentz transformations,

\[ \delta x^\mu = \xi^\mu(x) , \quad \delta q = (i/2) \omega_{ij} S^{ij} q , \] (2.3)

where \( \xi^\mu(x) \) are arbitrary four functions and \( \omega_{ij} \) are constant 6 parameters as before. Since we are now treating general coordinate transformations, we use Greek letters for coordinate indices and distinguish them from Lorentz indices denoted by Latin letters.

To meet with the invariance requirement, we must define those quantities \( \partial_k q \) which change under (2.3) in the same manner as \( \partial_k q \) of (2.2b). We define \( \partial_k q \) by

\[ \partial_k q = (\delta_k^\mu + c_k^\mu) \partial_\mu q , \] (2.4)

then we get the following transformation rule for \( c_k^\mu \);

\[ \delta c_k^\mu = \delta^\nu_c c_k^\nu + \omega^j_k c_j^\mu + \delta^\nu_c \omega^j_k + \delta^\mu_j \omega^-j_k \] (2.5)

The field \( c_k^\mu \) is the gauge field associated with the translation gauge group. It transforms inhomogeneously under extended translations. The special relativistic limit is obtained by putting \( c_k^\mu = 0 \). When \( c_k^\mu = 0 \), Greek indices are equivalent to Latin ones, and the transformations (2.3) compatible with (2.5) are restricted by

\[ \delta_j^\nu + \omega^\nu_j = 0 , \] (2.6)

from which we get (2.2a).

The transformation law of the translation gauge field given by (2.5) is rather complicated. The field \( b_k^\mu \) defined by

\[ b_k^\mu = \delta_k^\mu + c_k^\mu \] (2.7)

obeys much simpler transformation law,