1. INTRODUCTION

The theme of this meeting is the application of field theory to statistical mechanics. Of course such applications have previously been made in the context of particular physical problems. As examples we mention critical phenomena, the description of localization in disordered electronic systems, and soliton and exciton transport in molecular aggregates. In such studies the dynamical consequences of particular assumptions about the statistical properties of the system are explored. The statistics are often phenomenologically introduced by means of an equilibrium partition function for the field variables (order parameter). The source of the statistical fluctuations often remains obscure when this procedure is followed. Master equation approaches belong to this category.

Dynamic equations in which the fluctuations are explicitly displayed and analyzed have also been studied (stochastic differential equation approaches), but the specific physical mechanisms that result in particular statistical properties have usually not been adequately discussed. We focus on this topic here and consider the microscopic origins of fluctuations (and the associated dissipation) in thermodynamically closed systems. In our analysis we start from a Hamiltonian in which the system of interest, the heat bath, and the system–heat bath interaction are all treated dynamically. Our goal is to finally arrive at a stochastic differential equation of the generalized Langevin type for the system variables with completely determined statistical properties of the fluctuations. Our main emphasis will be on quantum systems. In particular we concentrate on two important effects: 1) the consequences of nonlinear interactions of the system with the surrounding heat bath, and 2) the consequences of the quantum mechanical nature of the system and of the heat bath. As we show below, the nonlinear interactions lead to interesting nonlinear deterministic and fluctuating potentials and to dissipative contributions in the stochastic equations, while
the quantum nature of the problem leads to fluctuation-dissipation relations that are distinctly different from their classical counterparts\textsuperscript{6,7}. Much of our recent research has been motivated by the desire to establish such connections between phenomenological formulations of stochastic equations and more fundamental dynamical approaches\textsuperscript{6-10}. In particular, we have had considerable success in establishing useful explicit relationships between model Hamiltonians and Langevin equations. We use the term "explicit" advisedly to indicate that our method is constructive as opposed to formal. One consequence of the present formulation of these connections is that it provides a prescription for determining the properties that a proper phenomenological model must have, be it quantum or classical. At this conference we wish to present some of the highlights of the applications of this approach.

2. CLASSICAL SYSTEMS

a. Linear Fluctuation-Dissipation Relations (Additive Fluctuations)

A massive particle in a fluid of light particles is subject to fluctuating and dissipative forces. The phenomenological description of the resulting "Brownian motion" is given by the well-known Langevin equation\textsuperscript{11}

\[ \dot{p} = -\lambda p + f(t) \] (2.1)

where \( p \) is the momentum of the (unit mass) massive particle. The coupling to the ambient fluid leads to the irreversible linear force \(-\lambda p\), where \( \lambda \) is a positive-definite coefficient related to the viscosity of the fluid. The coupling also produces reversible additive (i.e. \( p \)-independent) fluctuations \( f(t) \) about this average dissipative force. Langevin assumed the fluctuations to be Gaussian and delta-correlated in time,

\[ \langle f(t)f(t') \rangle = 2D\delta(t-t') \] (2.2)

Furthermore, Einstein's earlier results led Langevin to postulate a relation between the two forces so as to ensure the energy balance that leads to the thermal equilibration of the Brownian particle with the surrounding fluid. This fluctuation-dissipation relation (FDR) takes the form

\[ D = kT\lambda \] (2.3)

where \( T \) is the temperature of the fluid and \( k \) is Boltzmann's constant. We call (2.3) a linear FDR because it relates additive fluctuations and a linear dissipation, and recognize it as the relation that introduces the ambient temperature into the phenomenological theory. The above description is consistent with the usual notions of equilibrium statistical mechanics in that it leads to the canonical distribution

\[ P(p,t=\infty) \propto \exp(-p^2/2kT). \] (2.4)

A slight generalization of the above discussion to the Brownian motion of an oscillator of frequency \( \omega \) and unit mass (or to the Brownian