AN IMPROVED EULER METHOD FOR COMPUTING STEADY TRANSONIC FLOWS

D M Causon and P J Ford
Department of Aeronautical and Mechanical Engineering
University of Salford
Salford M5 4WT
England

SUMMARY
An improved Euler solver is presented for computing steady three-dimensional transonic flows around practical aircraft forebodies. The method is pseudo time-dependent, split and uses a finite-volume formulation. Shock waves are captured crisply, with minimal added smoothing, by means of an operator-switching facility which more accurately reflects the direction of propagation of signals. The method is robust, versatile, and holds promise for treating complex three-dimensional geometries economically.

INTRODUCTION
Computational aerodynamics is a revolutionary force in practical aerodynamic analysis as exemplified by the increasing number of solutions of compressible flow problems around complex configurations. With the advent of the fast, vectorised, Class 6 computers and associated increased storage capabilities, the feasibility of carrying out analyses of advanced three-dimensional aircraft configurations is becoming a reality. Pacing this advancement in computer technology is the development both of new algorithms and refinements of existing ones. The present approach is based upon the refinement of a method, which has proved to be simple to code and robust in applications to increasingly more complex and realistic configurations. In the development of this method, outlined below, the major emphasis has been on applicability and coding simplicity with the goal of providing a sound engineering tool for preliminary design and analysis. We have incorporated improvements to the method which can be proven in applications to yield clear benefits, but have avoided refinements which could result in large increases in code complexity with only slight improvements in the solutions. The resulting method shows promise for treating complex three-dimensional aircraft geometries economically. Recently [1,2] we described the method in some detail and presented a number of standard test-case solutions as evidence of the method's intrinsic validity. Here, we focus on applications of the method to some realistic aircraft forebodies in the upper transonic range of Mach numbers.

FORMULATION
Since a body-fitted mesh will be required, we cast the equations of motion in generalised co-ordinates. The Euler equations, in strong conservation form, are

$$\frac{\partial}{\partial t} \mathbf{U} + \frac{\partial}{\partial \mathbf{x}} \mathbf{F} = 0 \quad \mathbf{x} = 1(1)3$$

where $$\mathbf{U} = \mathbf{g}^T [\rho, \rho w_1, \rho w_2, \rho w_3, e]^T$$, $$\mathbf{F} = \mathbf{g}^{\mathbf{X}}$$, and $$\rho w_{1,2,3} u + p \delta x_{1,2,3}$$.

$$\mathbf{g}^{\mathbf{X}}$$ is the Jacobian and the flow velocity $$\mathbf{q} = u_{\mathbf{X}} = w_{\mathbf{X}} a_{\mathbf{X}}$$, where $$a_{\mathbf{X}}$$ are the Cartesian unit base vectors.
Equation (2) can also be cast in integral form, which is the basis of the finite volume method

$$\frac{\rho \mathbf{q}}{\partial t} + \int_{S_1} \mathbf{F} \cdot d\mathbf{s} + \int_{S_2} \mathbf{G} \cdot d\mathbf{s} + \int_{S_3} \mathbf{H} \cdot d\mathbf{s} = 0$$

The most appealing feature of the finite volume formulation becomes apparent by noting that $\frac{\partial \mathbf{q}}{\partial x_m} = g^k a_m$ and $\frac{\partial \mathbf{q} \cdot \mathbf{g}}{\partial x_m} = u^k$, where upon

$$\mathbf{F} = \mathbf{H}(\mathbf{U}) \cdot \mathbf{S}$$

and we see that computations can be performed with respect to the easily constructed Cartesian flux tensor $\mathbf{H}(\mathbf{U})$, rather than the curvilinear $\mathbf{F}'(\mathbf{U})$. Thus, we need not involve ourselves with the intricacies of the co-ordinate transformation.

**DISCRETISATION**

We solve equation (2) using a factored sequence of one-dimensional difference operators, where each component operator relates to its respective split differential equation

$$\frac{\rho \mathbf{q}}{\partial t} + \int_{S_1} \mathbf{F} \cdot d\mathbf{s} + \int_{S_2} \mathbf{G} \cdot d\mathbf{s} + \int_{S_3} \mathbf{H} \cdot d\mathbf{s} = 0 \quad \mathbf{s} = \text{either } 1, 2, \text{ or } 3$$

The MacCormack difference operator $L_1(\Delta t)$ is

$$\frac{\mathbf{U}^{n+1}}{\Delta t} = \frac{\mathbf{U}^n}{\Delta t} - \Delta t \left( \frac{\Delta - 1}{\Delta x^2} \right) \mathbf{U}^n$$

where $\Delta_+$ and $\Delta_-$ are respectively forward and backward two-point differences. In finite volume form, the operator $L_1(\Delta t)$ becomes

$$\mathbf{U}^{n+1} = \frac{1}{\Delta x} \left( \mathbf{U}^n - \Delta t \left( \frac{\Delta - 1}{\Delta x^2} \right) \mathbf{U}^n \right)$$

where $\Delta_+$ and $\Delta_-$ are respectively forward and backward two-point differences. In finite volume form, the operator $L_1(\Delta t)$ becomes

$$\mathbf{U}^{n+1} = \frac{1}{\Delta x} \left( \mathbf{U}^n - \Delta t \left( \frac{\Delta - 1}{\Delta x^2} \right) \mathbf{U}^n \right)$$

where $\mathbf{U}^{n+1}$ and $\mathbf{U}^n$ are the area vectors on opposite faces of the cell, corresponding to the surface $x^1 = \text{constant}$. Scheme (6) is easily coded, requiring only one level of storage and the area vectors and volumes can be evaluated from the Cartesian co-ordinates of the cell vertices [2].

The principal disadvantages of Scheme (6) are that it requires a numerical boundary condition at a supersonic exit, extra maxima and minima appear around shock waves, and additional smoothing is required around stagnation points, sonic lines and shocks. These negative features have been the motivation for much recent algorithm research and development. The present approach involves switching within a split operator between the MacCormack Scheme (6) and the upwind scheme of Beam and Warming [3], according to whether the flow is subsonic or supersonic. The upwind scheme, implemented in supersonic regions of flow, is

$$\mathbf{U}^{n+1} = \frac{1}{\Delta x} \left( \mathbf{U}^n - \Delta t \left( \frac{\Delta - 1}{\Delta x^2} \right) \mathbf{U}^n \right)$$