On Equivalence Transformations for Term Rewriting Systems

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Abstract

This paper proposes some simple methods, based on the Church-Rosser property, for testing the equivalence in a restricted domain of two reduction systems. Using the Church-Rosser property, sufficient conditions for the equivalence of abstract reduction systems are proved. These conditions can be effectively applied to test the equivalence in a restricted domain of term rewriting systems. In addition, equivalence transformation rules for term rewriting systems are proposed.

1. Introduction

The concept of the equivalence in a restricted domain of two term rewriting systems is presented here. The equivalence in a restricted domain means that the equational relation (or the transitive reflexive closure) generated by a reduction relation of one system is equal in a restricted domain to that of another system.

This concept plays an important role in transforming recursive programs [2][12] and proving an equation in abstract data types [3][5][6][9]. For example, consider a recursive program computing the factorial function on the set N of natural numbers represented by 0; S(0), S(S(0)), ...;

\[ F(x) = \text{IF equal}(x, 0) \text{ THEN } S(0) \text{ ELSE } x \times F(x-S(0)). \]

By using the successor function S, we can also define the factorial
function by:

\[ F(0) = S(0), \]
\[ F(S(x)) = S(x) \cdot F(x). \]

Regarding equations as rewriting rules from the left hand side to the right hand side, we can obtain two term rewriting systems \([4][5]\) from the above two definitions. The first term rewriting system can reduce "F(M)" to "IF equal(M,0) THEN S(0) ELSE M \cdot F(M-S(0))" for any term M, but the second system can not reduce "F(M)" unless M is either "0" or the form of "S(M')". Therefore the two term rewriting systems produce different results in the reduction of "F(M)", although they can reduce "F(M)" to the same result unless M can be reduced to a natural number. Thus, the equivalence for the recursive programs must be regarded as the equivalence in the restricted domain \(\mathbb{N}\) for the term rewriting systems.

We consider in this paper sufficient conditions for the equivalence in a restricted domain for two term rewriting systems. We first treat this problem in an abstract framework and show sufficient conditions for two abstract reduction systems. It is shown how one can formally validate the equivalence in the restricted domain for term rewriting systems by using these conditions. Finally, the problems related to the rules for transforming programs described by Burstall and Darlington \([2]\), and Scherlis \([12]\) are discussed, and equivalence transformation rules in a restricted domain for term rewriting systems are proposed.

2. Reduction Systems

We explain notions of reduction systems and give definitions for the following sections. These reduction systems have only an abstract structure, thus they are called abstract reduction systems \([4][7][11]\).

A reduction system is a structure \(R = \langle A, \rightarrow \rangle\) consisting of some object set \(A\) and some binary relation \(\rightarrow\) on \(A\), called a reduction relation. A reduction (starting with \(x_0\)) in \(R\) is a finite or infinite sequence \(x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow \ldots\). The identity of elements of \(A\)