A REPRESENTATION OF GRAPHS BY 
ALGEBRAIC EXPRESSIONS AND ITS 
USE FOR GRAPH REWRITING SYSTEMS

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Abstract: We define a set of operations on graphs and an algebraic notation for finite graphs. A complete axiomatization of the equivalence of graph expressions by equational rules is given. Graph rewriting systems can be defined as rewriting systems on graph expressions. This new definition is equivalent to the classical one using double push-outs.

Keywords: Many-sorted algebra, graph expression, graph rewriting system, graph-grammar.

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1 - Graphs and graph operations. 
(with applications to trees, series-parallel graphs and flowcharts)

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INTRODUCTION

Languages Theory deals with sets of words, with the various ways of defining them by grammars, automata and rewriting systems and with the numerous decision problems associated with these definitions.

Finite trees can be considered as generalized words (a word is a tree with only one branch i.e. with no "branching") so that similar questions can be raised for forests i.e. for sets of finite trees. The study of forests is considered as a part of formal languages theory.

The next natural extension is then to sets of finite graphs. Numerous works have already been devoted to graph grammars, graph rewriting systems and graph automata. See the bibliography by Nagl [21] (and the survey by Petrov [22] for graph automata). Applications are provided at length in the proceedings of the first two international workshops on graph-grammars [9,10].

One difficulty with graphs comes from the existence of many different definitions stemming from the various uses of graphs. In particular, there is no unique well-established notion of a context-free graph grammar as this is the case for languages. There are actually at least two ways to consider graphs:

- either as sets of vertices connected by edges (oriented or not, labeled or not etc...) and this is the point of view underlying the theory of NLC graph-grammars (Rozenberg et al. [19,25,26]),

- or as sets of edges (or hyperedges) glued together by means of vertices; this latter point of view is the one of Habel and Kreowski [16,17] to cite a few papers on context-free graph grammars, and also of the present work.

The original motivation of this work (i.e. of the set of results contained in [2] and [6]) was to develop a theory of context-free graph grammars as firmly grounded as the theory of context-free (word) grammars. But the outcome is actually much larger than the original aim as this survey and its companion [7] hope to show.

The two major features of context-free grammars are (in my opinion) the following ones:

(1) The existence of a derivation tree that represents the structure of derivation sequences up to some irrelevant permutations of derivation steps. This tree defines the analysis of the derived word w.r.t. the given grammar (or one possible analysis if the