Embedding Rule Independent Theory of Graph Grammars

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ABSTRACT. Recently the general theory of graph grammars has become a growing area of research. Some properties which hold for all sequential, vertex-replacing graph grammars (without erasing) are presented, including a vertex pumping lemma. A construction is described which proves the undecidability of the question whether a graph grammar has the following property: changing the order of application of the productions in a derivation does not change the graph produced. Classes of graph grammars for which this property can be decided are presented. They include the NLC graph grammars of Janssens and Rozenberg [4,5].

Key words: Sequential vertex-replacing graph grammars, general theory, order independence.

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1. INTRODUCTION

The study of graph grammars is an interesting and growing area of research. It has arisen from applications in computer science and biology, which yield a great diversity of problems and of classes of graph grammars used to solve them. For example, biological growth and parallel computations are both modelled using parallel graph grammars, while data structures and string grammars are modelled using sequential graph grammars. Authors have developed theoretical results for their particular classes of graph grammars, and have established when results for other classes were special cases of their own work.

In 1980 Janssens and Rozenberg began investigating the general theory of graph grammars by selecting a particular class (NLC grammars), and studying both its properties and the effects on these properties caused by extending and restricting the class [5]. Welzl [8] has proven results for an extremely general definition of a sequential, vertex-replacing graph grammar, which includes most of the classes of sequential, vertex-replacing graph grammars in the literature.

Such theoretical results are important in order to understand the power and limitations of graph grammars, to determine which applications they can model and which modifications of a graph gram-
mar will increase or decrease its generating power, and to establish guidelines for non-specialists who wish to apply these techniques.

Some new theoretical results for a general definition of graph grammars are sketched in this paper, along with decidability results for order independence. Full definitions and complete proofs will be found in the author’s Ph.D. thesis which is in progress.

2. PRELIMINARIES

Graph grammars have inherited an intrinsic graph theory difficulty, that of non-standard notation. This section defines the notation to be used throughout this article.

2.1. Graph Definitions

A vertex-labelled undirected loopless graph $G$ is a 4-tuple $G = (V, E, \Sigma, \varphi)$, where $V$ is a finite set of vertices, $E$ is a finite set of edges (pairs of distinct elements of $V$), $\Sigma$ is a set called the vertex-label alphabet, and $\varphi$ is a labelling function $\varphi : V \rightarrow \Sigma$. Henceforth vertex-labelled undirected loopless graphs are simply called graphs.

If $G = (V, E, \Sigma, \varphi)$ is a graph, and $U \subseteq V$, then the subgraph induced by $U$, denoted $G[U]$, is the graph with vertex set $U$, edge set $F = \{uv \mid uv \in E \text{ and } u, v \in U\}$, and labelling function $\varphi |_U$ into $\Sigma$.

If $\Delta$ is a finite set, and if $G = (V, E, \Sigma, \varphi)$ is a graph with $\Sigma \subseteq \Delta$, then $G$ is a graph over $\Delta$. The set of all graphs over $\Delta$ is denoted by $\mathcal{G}_\Delta$.

Two graphs $G = (V, E, \Sigma, \varphi)$ and $H = (V', E', \Sigma', \varphi')$ are said to be isomorphic, written $G \cong H$, if there exists a bijection $f$ from $V$ to $V'$ such that

$$uv \in E \iff f(u)f(v) \in E' \text{ and } \varphi(u) = \varphi'(f(u)) \text{ for all } u, v \in V.$$  

Such an $f$ is called a graph isomorphism.

2.2. Graph Grammar Definitions

The following definition of a graph grammar is general enough to include (essentially) all sequential, vertex-replacing graph grammars (possibly with some modifications) in the literature. (It is similar to the definitions used by Welzl [8], but does not allow what he calls application conditions.) Since other types of graph grammars will not be dealt with here, a sequential vertex-replacing graph grammar will simply be called a graph grammar.