Map OL systems with markers

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ABSTRACT. As an extension of binary, propagating map OL systems with markers, map OL systems with markers (mMOL systems) are introduced which are nondeterministic and also neither binary nor propagating. Although most OL systems have a positive solution for their membership problem, the mMOL systems do not have a positive solution for this decision problem. This recursive unsolvability is proved. Also, the generating power is shown.

Key words: map generating systems, map L systems, membership problem

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0. Introduction

Recently, there have been published a number of interesting papers on map generating systems. (For example, see papers in [1] and [2].) They also include the cell division systems which are motivated from a biological point of view.

A map is defined combinatorially by Tutte [3]. We give here a less formal definition of maps. A map is a finite connected planar graph in which multiple edges and loops are allowed. Furthermore, for every vertex in a map there exists a closed path to itself such that going in the clockwise direction from vertex to vertex always the rightmost different edge is chosen to form the path.
This definition implies the following:

1. A map consists of a finite set of regions (faces), which for biological reasons we call "walls".
2. Each wall is surrounded by a finite, circular sequence of edges (boundary).
3. One or more edges meet in a vertex.
4. Each edge has one or two vertices associated with it.
5. Edges cannot cross without forming a vertex.
6. There are no vertices without an associated edge.
7. Every edge is a part of the boundary of a wall.
8. Two walls can have in common one or more vertices and one or more edges of their boundaries.
9. There are no islands in walls.

Since the language theory on strings has been proposed, it has made surprising progress and now has established its proper research field. Meanwhile, the theory of two-dimensional languages is an active area as the fundamental theory of image processing. Further, graph grammars are also extremely interesting research topics with numerous applications.

In [4], we proposed two map generating systems based on string generation. The first one is binary, propagating map 0L system with markers (mBPM0L systems). The second one is binary, propagating map IL system with markers (mBPMIL systems). Then, we considered the following decision problems (1) and (2) on these systems:

1. Whether or not an arbitrary mBPM0L system is deterministic?
2. Whether or not an arbitrary mBPMIL system is deterministic?

These problems were shown to be decidable in paper [4]. Also, in the same paper the decision problem of stability of these two systems was solved. It is not difficult to show that the membership problem of propagating systems is recursively solvable.

In this paper, we shall consider an extension of binary, propagating map 0L systems with markers. We propose map 0L systems with markers (mM0L systems). The systems are nondeterministic, and also neither binary nor propagating. Then, we shall examine the membership problem of the mM0L systems. As an interesting result, we show that this decision problem is recursively unsolvable. The main purpose of this paper is to prove this fact. From this result, we know also that the generating power of mM0L systems is strictly stronger than that of mBPM0L systems.

We assume that readers are familiar with the basic definitions and notations of map generating systems.