COMPLEXITY OF PATTERN GENERATION VIA
PLANAR PARALLEL BINARY FISSION/FUSION GRAMMARS
(Abridged Version)

Jack W. Carlyle
Sheila A. Greibach
Computer Science Department
3731G Boelter Hall
School of Engineering and Applied Science
University of California, Los Angeles
Los Angeles CA 90024, USA

Azaria Paz
Department of Computer Science
Technion -- Israel Institute of Technology
Technion City, Haifa 32000 Israel

ABSTRACT. In two previous works [74CGP,86CGP], we defined and studied a scheme for
grammar-based generation of planar maps, using deterministic parallel (in the sense of Lindenmayer)
derivation rules of tow types — binary splitting of countries (fission) and binary merging (fusion). Coun-
tries would be visualized as cells in applications to modeling the development of (planar) biological organ-
isms, for example. We are concerned primarily with intrinsic properties of our model (rather than specific
applications), including its power as measured by the complexity of parallel generation of the class of all
legal (well-formed) planar maps. Our previous work showed that, with fission added to fusion, the com-
plexity decreases from linear to logarithmic in the size of the map. The present paper is a sequel, giving a
more complete treatment of several aspects of these basic results and related questions. In particular, we
examine: the effect of merging under grammar constraints; constructions involving Hamiltonian and semi-
Hamiltonian paths (tours of countries) and spanning trees (with countries as nodes), which are used as
intermediate phases in reaching the general logarithmic complexity result; and neighborhood (crowded-
ness) control in the generations.

Key words: cell-division, complexity, fission, fusion, Hamiltonian, linear time, logarithmic time,
pattern, planarity, graph, grammar, Lindenmayer system, map, parallel derivation.

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§1. INTRODUCTION. In two previous works [74CGP,86CGP], we defined and studied a scheme for grammar-based generation of planar maps. The grammatical rules are deterministic and of two types — splitting (fission) and merging (fusion) — and at each derivation step all countries are subject to rule application, i.e., derivations are parallel, and therefore rules interact and require reconciliation protocols. Splittings must be binary (suggested by biological cell division), and likewise mergers are restricted to be binary — pairwise combination via removal of a common border. Our motivation in formulating this model was to extend in a natural way to two dimensions the Lindenmayer systems introduced originally as a theoretical model for development of filamentous (essentially one-dimensional) biological organisms [68Li].

Other schemes have been proposed for generating graphs and related structures, including two-dimensional and three-dimensional structures. (Although we do not discuss it here, our model is capable of being extended to three-dimensional organisms. In either two or three dimensions, only neighbor relationships among cells are modeled, not Euclidean distances.) Space does not permit a comparative analysis, or a comprehensive reference list. Some references appear in [86CGP] and, e.g., a bibliography [83Na] of graph-related systems; recent references include [86NLA,86LL].

Fission and fusion have possible interpretations in data structures (fission/fusion correspond to specific primitives of the insertion/deletion variety) and in formal languages (fusion is a higher-dimensional generalization of erasing in string-based systems), aside from their significance in biological modeling, e.g., in explaining cell development of an organism (fission) and growth followed by differentiation (fission and fusion). At present we are concerned primarily with intrinsic properties of our model (rather than specific applications), including its power as measured by complexity of parallel generation of the class of all legal (well-formed) planar maps.

We have reported in [86CGP], in condensed fashion omitting some proofs, that the addition of fusion to fission reduces the worst-case number of parallel derivation steps for generation of a planar map (starting with an "egg" in our model) from linear to logarithmic in the size of the map. The present paper is a sequel, giving details omitted in [86CGP] and related results (to make the paper more self-contained, brief passages and illustrations have been repeated). Taken together with [86CGP], a more complete treatment of the generational complexity of the model is thus provided. In particular, in §5 through §8 we examine: the effect of merging under grammar constraints; constructions involving Hamiltonian and semi-Hamiltonian paths (tours of countries) and spanning trees (with countries as nodes), which are used as intermediate phases in reaching the general logarithmic complexity result; and neighborhood (crowdedness) control in the generations. We begin with a brief review of our model and of the complexity issues of interest, in §2, §3, and §4.