Abstract.

The main result of this paper is that a projection of the classical Hough transform for line detection onto a subspace of the parameter space (accumulator) will yield a useless trivial result if the composite operator consisting of projection and Hough transform is assumed to be linear and translation invariant.

1. Introduction

The Hough transform (or Radon transform) is an integral transform which assigns to a two-dimensional function $\beta(x,y)$ (the picture or image) the set of all integrals of this function along lines. In its classical formulation it was given by Radon (1917):

$$
H(\beta)(p, \alpha) := \int_{-\infty}^{\infty} \beta\left(p \cos \alpha + t \sin \alpha \right) dt.
$$

Hough (1962) made the observation that in binary images the maxima of this transformation correspond to lines in the image. In the meantime this concept was generalized by different researchers and it turned out to be a very useful tool in many applications of picture processing.

There are numerous attempts reported in the literature to reduce the dimension of the image space (the so-called accumulator) of the transform. There are two reasons for doing so:

The Hough transform assigns to each two-dimensional function a
two-dimensional function. Since the latter has to be calculated point by point, the computational effort for carrying out the transformation (the sampling effort) becomes large. The sampling effort can be reduced by data compression techniques. After the Hough transform has been calculated, a search for maxima becomes necessary. Also this search is far more easier in one dimension than in two dimensions. These arguments gain considerably weight if generalizations of the Hough transform are discussed whose image space has a dimension higher than two (Ballard 1981).

In the present paper the classical Hough transform for lines is investigated. It is shown that it is not possible to retain simultaneously three favourable properties when the Hough transform is projected onto a one-dimensional subspace of the parameter space:

- Linearity of the projection,
- Translation invariance of the projection,
- A nontrivial result of the projection.

This negative result can be generalized to a more general variant of the Hough transform, at the cost, however, of deeper functional analysis. Therefore, this generalization will be the subject of a separate investigation.

2. Basic Concepts

An image is a function \( B(P) \) associating to each point \( P=(x,y)^T \) of the plane a real number. We assume that the integral of the absolute value of \( B \) exists and that \( B \) vanishes outside a bounded set \( R \). Some examples for images are: binary images (\( B \) takes only values 0 and 1), gray tone images (\( 0 \leq B \leq 1 \)), gradient images (\( B \) is assumed to be the length of the gradient vector at point \( P \)). In the context of this paper we are mainly interested in binary images.

For analyzing and understanding the content of an image, the identification of certain features is important. In this paper we concentrate ourselves on linear features. For investigating the question whether a linear feature is contained in the image, we first parametrize all lines in the plane by number pairs \( (a,b) \). The set of all parameter pairs belonging to lines, the so-called