10. Shop problems with assignment

10.1 A hybrid flowshop problem with three stages

[Fortemps et al., 1996] are interested in a scheduling problem which occurs in the chemical industry, denoted by $HF3, (P6, P3, 1)|constr|\ F_\ell(C_{max}, \gamma(T_i), \delta(VPI))$, where *constr* translates a set of constraints described hereafter. The shop comprises three stages. Each job J_i is defined by a release date, and a due date. Each machine M_j also has an availability date, denoted by R_j.

At the first stage, six identical machines process the jobs which require setup times which are independent of the sequence on each machine (constraint $S^{(1)}_{nsd}$). Besides, the setup times require the intervention of a unique resource (a team of men), making it impossible to perform several preparations at the same time. This additional resource leads to the consideration that certain operations require several resources for their processing (constraint $fix_j^{(1)}$). When an operation is completed, the resource is freed up when the operation is ready to be processed at the second stage, that is to say once the transportation is finished. This constraint is denoted by $block^{(1,2)}$. Finally, each machine M_j has an unavailability time period, denoted by $unavail_j - resumable^{(1)}$, corresponding for example to a period for maintenance.
At the second stage, three identical machines can process the jobs after leaving the first stage. At this stage, splitting of jobs is authorised (constraint $split^{(2)}$). The first two stages are connected by a network of pipes. Several pipes are connected by valves. Use of a valve at a time t for the transportation of a job J_i makes useless every pathway via this valve for transportation of a job J_j, until transportation of J_i is finished. This constraint is denoted by $pipe^{(2)}$.
At the third stage, a single machine distributes the materials flow corresponding to each job to different receptacles.

The field of constraints in the notation of the problem can take the expression:
$constr = r_i^{(1)}, d_i^{(3)}, R_j, S^{(1)}_{nsd}, fix_j^{(1)}, block^{(1,2)}, pipe^{(2)}, unavail_j{-}resumable^{(1)}, split^{(2)}$

The aim is to compute a schedule that minimises a convex combination of three criteria. The criteria which are taken into account are:

1. the makespan, denoted by $C_{\max}$,
2. a penalty function $\gamma(T_1, \ldots, T_n)$ of job tardiness,
3. a penalty function $\delta(VPI)$ which reflects violation of periods of machines unavailability at the first stage. The presence of this criterion is due to the complexity of the problem which led the authors to relax the unavailability constraints.

Since the problem is $\mathcal{NP}$-hard, the algorithm proposed to solve it is an heuristic which proceeds in two phases. The first calculates a sequence L of jobs, which reflects the order in which the latter are introduced into the shop. The second phase does the calculation of the final schedule and the assignment of jobs on the resources at each stage.
Two heuristics are proposed to determine the sequence L. The first is a simulated annealing algorithm and the second is a tabu search. The determination of this list is made by looking for a solution which minimises the convex combination of the criteria. The general idea of the assignment heuristic is to schedule the jobs as soon as possible on the machines at each stage, according to the rule FAM. These assignments are made whilst respecting the constraints at each stage.

10.2 Hybrid flowshop problems with k stages

10.2.1 The $HFk, (PM^{(\ell)})_{\ell=1}^{k} || F_\ell(C_{max}, \overline{C})$ problem

[Riane et al., 1997] are interested in a scheduling problem where the shop has k stages and each stage ℓ contains $M^{(\ell)}$ identical machines. The aim is to minimise the makespan and the sum of completion times. For this, Riane, Meskens and Artiba minimise a convex combination of the criteria. This problem is $\mathcal{NP}$-hard.

Figure 10.1 presents a mixed linear integer program, denoted by ERMA1. Constraints (A) express the fact that the jobs must be processed at every stage. Constraints (B) imply that there is at most one job in position ℓ on each machine. Constraints (C) and (D) enable us to calculate the completion times at each stage (both routing and disjunctive constraints on the machines). Finally, constraints (E) and (F) define the criteria $C_{\max}$ and $\overline{C}$ to be minimised.

Two tabu search heuristics are also proposed. The first randomly generates a sequence of jobs. Assignment at the first stage is made according to the rule FAM and the jobs are scheduled as soon as possible. We suppose that the assignments are made at the following stages using the same rule and by