

3. Multicriteria optimisation theory

3.1 MCDA and MCDM: the context

Decision Making arises at all levels in firms. A firm may be described as a “complex system”, and we can make the following remarks ([Boldur, 1982]):

- A complex system can be broken down into sub-systems according to the objectives of the first one (production sub-system, human resources management sub-system, *etc.*).
- The methods of management must be arranged in order to propose solutions that fit the actual objectives.
- It is necessary to mix different disciplines such as Operational Research, Management and Psychology in order to thoroughly understand and model a complex system.

These remarks make apparent the complexity of the decision processes in the firms. The desire to rationalise these processes to the extreme leads inevitably to an aberration ([Roy, 1985]) as certain factors, which occur in real time, cannot be taken into account in advance. The multicriteria decision domain proposes a set of tools which enables to model the decision process more or less faithfully ([Boldur, 1982]).

The representation of the decision process, or even simply the search for a correct decision, is conditioned by different elements ([Zionts, 1997]):

- Well defined decisions do not exist all the time, sometimes only “orientations” exist.
- The decision maker is rarely a unique individual. Often there is a group of people that take decisions.
- The set of possible decisions (or actions, or alternatives) is rarely fixed, but tends to evolve in real time.
- Although the decision maker wants to choose the optimal decision, this perhaps does not exist or else he is incapable of differentiating between a good decision and the optimal solution.

Two particular domains, *MultiCriteria Decision Making* and *MultiCriteria Decision Aid*, are found in the literature. The difference lies mainly in the way to model the problems.

3.1.1 MultiCriteria Decision Making

MultiCriteria Decision Making (MCDM), is a *descriptive* approach (see [Roy and Bouyssou, 1993] and [Roy, 1990]) as it consists of describing the problem:

- by defining the possible decisions,
- by defining the attributes (the consequences of these decisions) and the evaluation criteria,
- by incorporating in a utility function f the set of retained criteria.

Finally, we choose the decision which maximises this function. This approach is based on a certain number of fundamental axioms ([Roy, 1985] and [Roy and Bouyssou, 1993]):

- When the decision maker makes a decision he maximises, implicitly or explicitly, a utility function.
- An optimal decision exists in every situation.
- Two decisions which might be incomparable do not exist. We can make a choice or a sort between every pair of decisions.
- Formally, the decision maker's preferences hinge upon two binary relations: the preference P and the indifference I . Let us consider two decisions a and b , either a is preferable to b (aPb), or b is preferable to a (bPa) or a and b are indifferent (aIb). These two relationships are transitive.

Different methods classified in the MCDM approach exist (see for example [Olson et al., 1997] and [Guitouni and Martel, 1997]). Among the best known and most used are:

- the methods relating to the MultiAttribute Utility Theory (MAUT, see [VonNeumann and Morgenstern, 1954]) which use a stochastic approach. These methods concern the problems where the different decisions are subject to uncertainty at the criteria level. Finally, these approaches assume that the decision maker is alone.
- the Analytic Hierarchy Process method (AHP, [Saaty, 1986]), which classifies the criteria into groups using a hierarchical analysis in the form of a tree. Each criterion has a weight inside the objective function and to fix these weights the decision maker must compare each pair of criteria and he must give a ratio that reflects his preference.

[Dyer et al., 1992] give a critical presentation of the works in the MultiCriteria Decision Making domain and break it down as shown in figure 3.1.

3.1.2 MultiCriteria Decision Aid

MultiCriteria Decision Aid (MCDA) is an approach known as *constructive* ([Roy and Bouyssou, 1993] and [Roy, 1990]). It does not seek an optimal