2. An Introduction to the Incremental-Iterative Solution of Nonlinear Structural Problems

This chapter gives an introduction to nonlinear structural analysis. The material selected gives some basic solution procedures and provides the fundamental relations of incremental nonlinear analysis.

We first review in Section 2.1 some commonly used solution procedures for one nonlinear equation and then present the Newton-Raphson method for a set of nonlinear equations. In Section 2.2 we introduce how we solve nonlinear structural problems through a simple example, and then show how the Newton-Raphson method is used for general finite element systems.

Section 2.3 is devoted to the linearization of the principle of virtual work equation for general nonlinear deformations of a material body and the derivation of the incremental-iterative equations. The finite element discretization of a solid continuum using the updated Lagrangian (UL) formulation is considered. The element matrices for the three-dimensional isoparametric solid finite element are derived.

The presentation given here introduces the reader to the overall solution process of nonlinear structural analysis, such that the context and details of the inelastic solution algorithms presented in the next chapters can be understood. More details of nonlinear finite element analysis using the same notation are given in Bathe (1996).

2.1 Some Solution Procedures for Nonlinear Equations

In this section we briefly present some solution procedures for one nonlinear equation and for a set of nonlinear equations.

2.1.1 Some Solution Procedures for One Nonlinear Equation

Bisection Method. A simple and robust solution procedure is the bisection method, or method of halving the interval. The procedure consists of the following steps.

Suppose that a nonlinear equation

$$\boxed{f(x) = 0} \tag{2.1.1}$$

has only one solution in the interval $[a, b]$ and that $f(x)$ is a continuous function in this interval. Assume that $f(b) = f_b > 0$ and $f(a) = f_a < 0$, as schematically shown in Fig. 2.1.1. We start the procedure by calculating the trial solution x_0 as

$$x_0 = \frac{1}{2}(a + b) \tag{2.1.2}$$

If $f(x_0) = f_0 > 0$ we use the new interval $[a, x_0]$ and calculate

$$x_1 = \frac{1}{2}(a + x_o) \tag{2.1.3}$$

as shown in the figure. If f_0 were negative we would have used the interval $[x_0, b]$ as the new interval containing the zero of $f(x)$, and $x_1 = 0.5(x_0 + b)$. Denoting by x_i^+ and x_i^- the values of x with positive and negative function values of $f(x)$, respectively, and the i-th interval by $\left[x_i^-, x_i^+\right]$ we can write the relation

$$\boxed{x_i^+ - x_i^- = \frac{1}{2}(x_{i-1}^+ - x_{i-1}^-)} \tag{2.1.4}$$

The bisection procedure stops when the size of the interval is less than a given tolerance ϵ, i.e., when

$$\mid x_i^+ - x_i^- \mid < \epsilon \tag{2.1.5}$$

The error estimate is then

$$\boxed{\mid x_i - x^* \mid \leq \frac{1}{2^{i+1}}(b - a)} \tag{2.1.6}$$

where x^* is the solution of $f(x) = 0$.

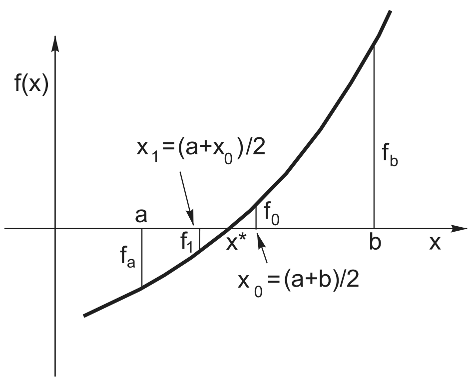

Fig. 2.1.1. Bisection procedure to solve the equation $f(x) = 0$