3. Fundamental Notions of Metal Plasticity

In this chapter we present fundamentals of the theory of plasticity with the view towards the development of a general numerical procedure for the stress calculation introduced and used in the subsequent chapters. After a brief introduction, we give in Section 3.2 the basic notions of plasticity and define the von Mises material model. Then, in Section 3.3 we present some commonly used orthotropic metal plasticity models. In each section we illustrate the theoretical concepts presented by means of various examples.

3.1 Introduction

The development and application of theories of plasticity to engineering problems started with the pioneering works of Tresca (1864); St. Venant (1870); Levy (1870); followed by seminal contributions of von Mises (1913); Prandtl (1924) and Reuss (1930). A detailed presentation of the history of strength of materials is presented in Timoshenko (1953). Today, the use of plasticity in the engineering disciplines is well established. In general, the theories of plasticity can be divided into two categories: *micromechanical theories* and *macromechanical theories*. The micromechanical theories analyze the plastic deformations on the microscopic level and seek to explain the conditions in crystals and grains of metals leading to plastic flow, e.g., Rice (1971, 1975); Asaro (1983); Aifantis (1987).

On the other hand, the macromechanical theories (also called the mathematical theories) of plasticity describe plastic deformations phenomenologically, on the macroscopic level, and establish relations among the macroscopic mechanical quantities (such as stresses, strains, etc.). These relations are based on general principles of mechanics and on experimental observations. The fundamentals of macromechanical theories of plasticity are given in many books, such as those of Hill (1950); Prager (1959); Prager and Hodge (1968); Mendelson (1968); Życzkowski (1981); Chen and Saleeb (1982); Chen and Han (1988); Lubliner (1990); Ulm and Coussy (2003). A unification of the macromechanical theories of inelastic material behavior, named disturbed state concept, is presented in Desai (2001).

Many practical problems accounting for the plastic deformations of materials have been successfully solved. Some early solution methods are based

on variational theorems developed mainly in the fifth decade of the previous century, largely due to Drucker, Prager and Hill (see References). These variational solutions provide upper and lower-bound theorems for the ultimate load capacity of structures. The load carrying capacity of specimen can also be calculated using the method of characteristics, see for example Hill (1950); Prager (1955, 1956). With these classical methods, it is hard or even impossible to obtain an elastic-plastic solution which gives the total *history of deformation*, from the elastic to the ultimate load state.

In today's engineering environment, it is imperative that detailed analysis of geometrically very complicated structures be carried out. These analyses must trace out the complete response history from elastic to plastic conditions, including the progression of elasto-plasticity and large deformations, until possible collapse of the structure.

The aim of this chapter is to review fundamental notions of metal plasticity in order to establish the basis for introducing robust numerical procedures. We consider the solution of problems of elasto-plasticity modeled using classical macromechanical theories of plasticity, and assume that finite element solutions are sought.

The computational algorithms to be presented in the subsequent chapters correspond to the so-called *strain-driven* methods. Namely, we shall employ an incremental solution process (outlined in the previous chapter) in which we consider that the total strains at a material point are known at a certain time (load) step; and that the stresses corresponding to the given strains need to be calculated. This procedure is generally used in displacement-based or mixed finite element discretizations (see Chapter 2). Throughout the presentations below we follow the notation used in Bathe (1996).

3.2 Isotropic Plasticity

The classical macromechanical theories of plasticity are based on the notions of a yield surface or yield function giving the yield condition, a hardening rule (governing the change of the yield surface during deformations) and on the stress-plastic strain relations of the material.

These notions are used to formulate a material model for the calculation of the material response during plastic deformations. In the following presentation of these fundamental notions, we adopt two approaches (Sections 3.2.1 to 3.2.4).

- We start from experimental observations and give the mathematical relations to model these observations; or
- We establish mathematical relations based on a mechanical principle, and present experimental results that confirm these relations.

The fundamental relations are then employed for the definition of the von Mises material model in Section 3.2.5, in a form generally accepted for