Market LIBOR Models

As was mentioned already, the acronym LIBOR stands for the London Interbank Offered Rate. It is the rate of interest offered by banks on deposits from other banks in eurocurrency markets. Also, it is the floating rate commonly used in interest rate swap agreements in international financial markets (in domestic financial markets as the reference interest rate for a floating rate loans it is customary to take a prime rate or a base rate). LIBOR is determined by trading between banks and changes continuously as economic conditions change. For more information on market conventions related to the LIBOR and Eurodollar futures, we refer to Sect. 9.3.4.

In this chapter, we present an overview of recently developed methodologies related to the arbitrage-free modelling of market rates, such as LIBORs. In contrast to more traditional aproaches, term structure models developed recently by, among others, (Miltersen et al. 1997), Brace et al. (1997), Musiela and Rutkowski (1997), Jamshidian (1997a), Hunt et al. (1996, 2000), Hunt and Kennedy (1996, 1997, 1998), and Andersen and Andreasen (2000b), are tailored to handle the most actively traded interest-rate options, such as caps and swaptions. For this reason, they typically enjoy a higher degree of tractability than the classical term structure models based on the diffusion-type behavior of instantaneous (spot or forward) rates.

Recall that the Heath-Jarrow-Morton methodology of term structure modelling is based on the arbitrage-free dynamics of instantaneous, continuously compounded forward rates. The assumption that instantaneous rates exist is not always convenient, since it requires a certain degree of smoothness with respect to the tenor (i.e., maturity) of bond prices and their volatilities. An alternative construction of an arbitrage-free family of bond prices, making no reference to the instantaneous, continuously compounded rates, is in some circumstances more suitable.

The first step in this direction was taken by Sandmann and Sondermann (1993b), who focused on the effective annual interest rate (cf. Sect. 10.1.2). This idea was further developed by Goldys et al. (2000), Musiela (1994), Sandmann et al. (1995), Miltersen et al. (1997) and Brace et al. (1997).

It is worth pointing out that in all these papers, the HJM framework is adopted (at least implicitly). For instance, Goldys et al. (2000) introduce a HJM-type model based on the rate $j(t, T)$, which is related to the instantaneous forward rate through
the formula \( 1 + j(t, T) = e^{f(t, T)} \). The model put forward in this paper assumes a deterministic volatility function for the process \( j(t, T) \). A slightly more general case of nominal annual rates \( q(t, T) \), which satisfy (\( \delta \) representing the duration of each compounding period)

\[
(1 + \delta q(t, T))^{1/\delta} = e^{f(t, T)},
\]

was studied by Musiela (1994), who assumes the deterministic volatility \( \gamma(t, T) \) of each nominal annual rate \( q(t, T) \). This implies the following form of the coefficient \( \sigma \) in the dynamics of the instantaneous forward rate

\[
\sigma(t, T) = \delta^{-1}(1 - e^{-\delta f(t, T)})\gamma(t, T),
\]

so that the model is indeed well-defined (that is, instantaneous forward rates, and thus also the nominal annual rates, do not explode). Unfortunately, these models do not give closed-form solutions for zero-coupon bond options, and thus a numerical approach to option pricing is required. Miltersen et al. (1997) focus on the actuarial (or effective) forward rates \( a(t, T, U) \) satisfying

\[
(1 + a(t, T, T + \delta))^\delta = \exp\left(\int_T^{T+\delta} f(t, u) \, du\right).
\]

They show that a closed-form solution for the bond option price is available when \( \delta = 1 \). More specifically, an interest rate cap is priced according to the market standard. However, the model is not explicitly identified and its arbitrage-free features are not examined, thus leaving open the question of pricing other interest rate derivatives. These problems were addressed in part in a paper by Sandmann et al. (1995), where a lognormal-type model based on an add-on forward rate (add-on yield) \( f_s(t, T, T + \delta) \), defined by

\[
1 + \delta f_s(t, T, T + \delta) = \exp\left(\int_T^{T+\delta} f(t, u) \, du\right),
\]

was analyzed. Finally, using a different approach, Brace et al. (1997) explicitly identify the dynamics of all rates \( f_s(t, T, T + \delta) \) under the martingale measure \( \mathbb{P}^* \) and analyze the properties of the model.

Let us summarize the content of this chapter. We start by describing in Sect. 12.1 forward and futures LIBORs. The properties of the LIBOR in the Gaussian HJM model are also dealt with in this section. Subsequently, in Sect. 12.4 we present various approaches to LIBOR market models. Further properties of these models are examined in Sect. 12.5. In Sect. 12.2 we describe interest rate cap and floor agreements. Next, we provide in Sect. 12.3 the valuation results for these contracts within the framework of the Gaussian HJM model. In Sect. 12.6, we deal with the valuation of contingent claims within the framework of the lognormal LIBOR market model. In the last section, we present briefly some extensions of this model.