In this chapter, we deal with derivative securities related to at least two economies (a domestic market and a foreign market, say). Any such security will be referred to as a cross-currency derivative. In contrast to the model examined in Chap. 4, all interest rates and exchange rates are assumed to be random. It seems natural to expect that the fluctuations of interest rates and exchange rates will be highly correlated. This feature should be reflected in the valuation and hedging of foreign and cross-currency derivative securities in the domestic market. Feiger and Jacquillat (1979) (see also Grabbe (1983)) were probably the first to study, in a systematic way, the valuation of currency options within the framework of stochastic interest rates (they do not provide a closed-form solution for the price, however). More recently, Amin and Jarrow (1991) extended the HJM approach by incorporating foreign economies. Frachot (1995) examined a special case of the HJM model with stochastic volatilities, in which the bond price and the exchange rate are assumed to be deterministic functions of a single state variable.

The first section introduces the basic assumptions of the model along the same lines as in Amin and Jarrow (1991). In the next section, the model is further specified by postulating deterministic volatilities for all bond prices and exchange rates. We examine the arbitrage valuation of foreign market derivatives such as currency options, foreign equity options, cross-currency swaps and swaptions, and basket options (see Jamshidian (1988, 1994a), Turnbull (1994), Frey and Sommer (1996), Brace and Musiela (1997), Dempster and Hutton (1997), Mikkelsen (2002), and Schlögl (2002)).

Let us explain briefly the last three contracts. A cross-currency swap is an interest rate swap agreement in which at least one of the reference interest rates is taken from a foreign market; the payments of a cross-currency swap can be denominated in units of any foreign currency, or in domestic currency. As one might guess, a cross-currency swaption is an option contract written on the value of a cross-currency swap. Finally, by a basket option we mean here an option written on a basket (i.e., weighted average) of foreign interest rates. Typical examples of such contracts are basket caps and basket floors.
The final section is devoted to the valuation of foreign market interest rate derivatives in the framework of the lognormal model of forward LIBOR rates. It appears that closed-form expressions for the prices of such interest rate derivatives as quanto caps and cross-currency swaps are not easily available in this case, since the bond price volatilities follow stochastic processes.

14.1 Arbitrage-free Cross-currency Markets

To analyze cross-currency derivatives within the HJM framework, or in a general stochastic interest rate model, we need to expand our model so that it includes foreign assets and indices. Generally speaking, the superscript \(i\) indicates that a given process represents a quantity related to the \(i^{th}\) foreign market. The exchange rate \(Q^i_t\) of currency \(i\), which is denominated in domestic currency per unit of the currency \(i\), establishes the direct link between the spot domestic market and the \(i^{th}\) spot foreign market. As usual, we write \(\mathbb{P}^*\) to denote the domestic martingale measure, and \(W^*\) stands for the \(d\)-dimensional standard Brownian motion under \(\mathbb{P}^*\). Our aim is to construct an arbitrage-free model of foreign markets in a similar way to that of Chap. 4. In order to avoid rather standard Girsanov-type transformations, we prefer to start by postulating the “right” (that is, arbitrage-free) dynamics of all relevant processes. For instance, in order to prevent arbitrage between investments in domestic and foreign bonds, we assume that the dynamics of the \(i^{th}\) exchange rate \(Q^i_t\) under the measure \(\mathbb{P}^*\) are

\[
dQ^i_t = Q^i_t((r^i - r^i_t)\, dt + \nu^i_t \cdot dW^*_t), \quad Q^i_0 > 0,
\]

where \(r_t\) and \(r^i_t\) stand for the spot interest rate in the domestic and the \(i^{th}\) foreign market, respectively. The rationale behind expression (14.1) is similar to that which leads to formula (4.14) of Chap. 4 (see also formulas (14.8)–(14.9) below). In the cross-currency HJM approach, the interest rate risk is modelled by the domestic and foreign market instantaneous forward rates, denoted by \(f(t, T)\) and \(f^i(t, T)\) respectively. We postulate that for any maturity \(T \leq T^*\), the dynamics under \(\mathbb{P}^*\) of the foreign forward rate \(f^i(t, T)\) are

\[
df^i(t, T) = \sigma^i(t, T) \cdot (\sigma^*_i(t, T) - \nu^i_t)\, dt + \sigma^i(t, T) \cdot dW^*_t,
\]

where

\[
\sigma^*_i(t, T) = \int_t^T \sigma_i(t, u)\, du, \quad \forall t \in [0, T].
\]

We assume also that for every \(i\) we are given an initial foreign term structure \(f^i(0, T), T \in [0, T^*]\), and that the foreign spot rates \(r^i_t\) satisfy \(r^i_t = f^i(t, t)\) for every \(t \in [0, T^*]\). The price \(B^i(t, T)\) of a \(T\)-maturity foreign zero-coupon bond, denominated in foreign currency, is

\[
B^i(t, T) = \exp\left(-\int_t^T f^i(t, u)\, du\right), \quad \forall t \in [0, T].
\]