Efficient 1-Bit-Communication Cellular Algorithms

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Summary. We propose several efficient algorithms for a large scale of cellular automata having 1-bit inter-cell communications (CA\(_{1\text{-bit}}\)). A 1-bit inter-cell communication model studied in this paper is a new class of cellular automata (CA) whose inter-cell communication is restricted to 1-bit. We call the model 1-bit CA in short. The number of internal states of the 1-bit CA is assumed to be finite in a usual way. The next state of each cell is determined by the present state of itself and two binary 1-bit inputs from its left and right neighbor cells. Thus the 1-bit CA can be thought to be one of the most powerless and simplest models in a variety of CAs.

We study a sequence generation problem, a firing squad synchronization problem and an early bird problem, all of which are known as the classical and fundamental problems in cellular automata.

First we consider the sequence generation problem. It is shown that there exists a 1-state CA\(_{1\text{-bit}}\) that can generate in real-time a context-sensitive sequence such that \(\{2^n|n=1,2,3,...\}\). Prime sequence can also be generated in real-time by CA\(_{1\text{-bit}}\) with 34 states. Secondary, we study the firing squad synchronization problem on two-dimensional CA\(_{1\text{-bit}}\). We give a two-dimensional CA\(_{1\text{-bit}}\) which can synchronize any \(n \times n\) square and \(m \times n\) rectangular arrays in \(2n-1\) and \(m+n+\max(m,n)\) steps, respectively. In addition, we propose a generalized synchronization algorithm that operates in linear steps on two-dimensional rectangular arrays with the general located at an arbitrary position of the array. The time complexities for the first two algorithms developed are one to two steps larger than optimum ones proposed for O(1)-bit communication model. In the last, we give a 1-bit implementation for an early bird problem. It is shown that there exists a 12-state CA\(_{1\text{-bit}}\) that solves the early bird problem in linear time.

1 Introduction

In recent years cellular automata (CA) have been establishing increasing interests in the study of modeling real phenomena occurring in biology, chemistry,
ecology, economy, geology, mechanical engineering, medicine, physics, sociology, public traffic, etc. Cellular automata are considered to be a good model of complex systems in which an infinite one-dimensional array of finite state machines (cells) updates itself in synchronous manner according to a uniform local rule.

In this paper, we study a sequence generation problem [1, 4, 7, 19, 20], a firing squad synchronization problem [2, 5, 6, 10-13, 15-17, 21-25] and an early bird problem [3, 8, 9, 14, 23], all of which are known as the classical and fundamental problems studied extensively on O(1)-bit communication models of cellular automata. An O(1)-bit communication model is a conventional CA where the amount of communication bits exchanged at one step between neighboring cells is assumed to be O(1)-bit, however, such bit-information exchanged between inter-cells has been hidden behind the definition of conventional automata-theoretic finite state descriptions. On the other hand, a 1-bit inter-cell communication model studied in this paper is a new CA whose inter-cell communication is restricted to 1-bit. We call the model 1-bit CA in short, and it is denoted as CA$_{1\text{-bit}}$. The number of internal states of the 1-bit CA is assumed to be finite in a usual way. The next state of each cell is determined by the present state of itself and two binary 1-bit inputs from its left and right neighbor cells. Thus the 1-bit CA can be thought to be one of the most powerless and simplest models in a variety of CAs.

In the next section 2, we define formally a 1-bit communication cellular automaton (CA$_{1\text{-bit}}$) and gives a computational relation between the conventional CA and the CA$_{1\text{-bit}}$. In section 3, we consider a sequence generation problem on CA$_{1\text{-bit}}$ and give several non-regular sequences that can be generated in real-time by CA$_{1\text{-bit}}$. In section 4, a synchronization problem is studied and three 1-bit implementations of synchronization algorithms for two-dimensional square and rectangular arrays will be given. In the last section, an early bird problem is considered and an efficient 12-state implementation will be given. Due to the space available, we omit the details of the proofs of theorems given below.

2 One-Bit Communication Cellular Automata

A one-dimensional 1-bit inter-cell communication cellular automaton [13, 18-20] consists of an infinite array of identical finite state automata, each located at positive integer point. Each automaton is referred to as a cell. A cell at point $i$ is denoted by $C_i$ where $i \geq 1$. Each $C_i$, except $C_1$, is connected with its left and right neighbor cells via a left or right one-way communication link, where those communication links are indicated by right- and left-going arrows, as is shown in Fig. 1, respectively. Each one-way communication link can transmit only one bit at each step in each direction. One distinguished leftmost cell $C_1$, the communication cell, is connected to outside world. A cellular automaton