7 Heston’s Model and the Smile

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7.1 Introduction

The Black-Scholes formula, based on the assumption of log-normal stock diffusion with constant volatility, is the universal benchmark for option pricing. But as all market participants are keenly aware of, it is flawed. The model-implied volatilities for different strikes and maturities of options are not constant and tend to be smile shaped. Over the last two decades researchers have tried to find extensions of the model in order to explain this empirical fact.

A very natural approach, suggested already by Merton (1973), allows the volatilities to be a deterministic function of time. While it explains the different implied volatility levels for different times of maturity, it still does not explain the smile shape for different strikes. Dupire (1994), Derman and Kani (1994), and Rubinstein (1994) came up with the idea of allowing not only time, but also state dependence of the volatility coefficient, see Fengler (2005) and Chapter 6. This local (deterministic) volatility approach yields a complete market model. Moreover, it lets the local volatility surface to be fitted, but it cannot explain the persistent smile shape which does not vanish as time passes.

The next step beyond the local volatility approach was to allow the volatility coefficient in the Black-Scholes diffusion equation to be random. The pioneering work of Hull and White (1987), Stein and Stein (1991), and Heston (1993) led to the development of stochastic volatility models. These are two-factor models with one of the factors being responsible for the dynamics of the volatility coefficient. Different driving mechanisms for the volatility process have been proposed, including geometric Brownian motion and mean-reverting Ornstein-Uhlenbeck type processes.
Heston’s model stands out from this class mainly for two reasons: (i) the process for the volatility is non-negative and mean-reverting, which is what we observe in the markets, and (ii) there exists a closed-form solution for vanilla options. It was also one of the first models that was able to explain the smile and simultaneously allow a front-office implementation and a market consistent valuation of many exotics. Hence, we concentrate in this chapter on Heston’s model. First, in Section 7.2 we discuss the properties of the model, including marginal distributions and tail behavior. In Section 7.3 we adapt the original work of Heston (1993) to a foreign exchange (FX) setting. We do this because the model is particularly useful in explaining the volatility smile found in FX markets. In equity markets the typical volatility structure is an asymmetric skew (also called a smirk or grimace). Calibrating Heston’s model to such a structure leads to very high, unrealistic values of the correlation coefficient. Finally, in Section 7.4 we show that the smile of vanilla options can be reproduced by suitably calibrating the model parameters.

However, we do have to say that Heston’s model is not a panacea. The criticism that we might want to put forward is that the market consistency could potentially be based on a large number of market participants using it! Furthermore, while trying to calibrate short term smiles, the volatility of volatility often seems to explode along with the speed of mean reversion. This is a strong indication that the process “wants” to jump, which of course it is not allowed to do. This observation, together with market crashes, has lead researchers to consider models with jumps. Interestingly, jump-diffusion models have been investigated already in the mid-seventies (Merton, 1976), long before the advent of stochastic volatility. Jump-diffusion models are, in general, more challenging to handle numerically than stochastic volatility models. Like the latter, they result in an incomplete market. But, whereas stochastic volatility models can be made complete by the introduction of one (or a few) traded options, a jump-diffusion model typically requires the existence of a continuum of options for the market to be complete.

Recent research by Bates (1996) and Bakshi, Cao, and Chen (1997) suggests using a combination of jumps and stochastic volatility. This approach allows for even a better fit to market data, but has so many parameters, that it is hard to believe that there is enough information in the market to calibrate them. Andersen and Andreasen (2000) let the stock dynamics be described by a jump-diffusion process with local volatility. This method combines ease of modeling steep short-term volatility skews (jumps) and accurate fitting to quoted option prices (deterministic volatility function). Other alternative approaches utilize Lévy processes (Barndorff-Nielsen, Mikosch, and Resnick, 2001; Eberlein,