In this paper, the physical principles of local electrical observations and measurements and related analytical formulas, as well as the optimal experimental conditions for electrical measurements, are first reminded. Secondly, it is shown, based on experiments and analytical expressions, how the observations obtained with the methods of force and force-gradient microscopy can be analysed. The advantage of the force-gradient microscopy over the force microscopy in better resolution and precision of measurements is demonstrated. The related analytical formulas are given, which explain the advanced capabilities of the force-gradient microscopy for precise electrical studies on local objects on the subnanometer scale. Finally, based on recent published data, the main applications of the electrostatic-force and force-gradient microscopy for analysis of materials and devices are briefly considered.

### Abbreviations

- $a$: damping
- $A_M$: maximum mechanical amplitude
- $A_\Omega$: maximum electrical amplitude at pulsation $\Omega$
- $A_{2\Omega}$: maximum electrical amplitude at pulsation $2\Omega$
- $b$: coefficient
- $C$: tip to sample capacitance which is composed of two capacitors in series: the tip to surface plus the surface to bulk capacitors. First and second derivatives of the tip to sample capacitance in the direction perpendicular to the surface, or capacitance coupling capacitance couplings with an infinite conducting surface and the apex, cone and cantilever parts of the sensor

$C' = \partial C/\partial z$, $C'' = \partial^2 C/\partial z^2$

- $C'_{\text{apex}}$, $C'_{\text{cone}}$, $C'_{\text{canti}}$, $C''_{\text{apex}}$, $C''_{\text{cone}}$, $C''_{\text{canti}}$: capacitance coupling with the area of interest (centre of a track or disk) or, in abbreviation, $C'_j$, $j = 1$ or $2$, respectively
- $C'_{1}$, $C'_{1}$: capacitance coupling outside the area of interest (centre of a track or disk) or, in abbreviation, $C'_j$, $j = 1$ or $2$, respectively
small capacitance coupling variations
small variations of force or force gradient dc or at pulsations $\Omega$ or $2\Omega$, respectively

small surface voltage variations
local sphere to sample distance
diameter of a disk on sample surface totally or partially coupled electrically with the apex surface

Parts of the electrical force applied on the apex, cone and cantilever, respectively

mechanical force
electrical force normal to the sample surface
dc electrical force
force at electrical pulsation $\Omega$
force at the double of the electrical pulsation $\Omega$

first derivative of the electrical force in the direction of vibration (perpendicularly to the sample surface)
first derivative of the electrical forces (dc, $\Omega$, $2\Omega$ respectively), in the direction of vibration ($\perp$ to the sample surface)
tip cone height
sensor stiffness
ratio between apex and cone capacitance couplings

ratio between measured and real voltage differences
sensor effective mass
local electrostatic pressure
quality coefficient of the resonance
local surface charge
tip apex radius
lateral resolution obtained in force or force gradient
tip to sample voltage due to work function differences (and sample polarisation)
dc tip to sample voltage externally applied
efficient dc voltage
ac tip to sample voltage externally applied (pulsation $\Omega$)
mean tip to sample distance
sum of the different amplitudes of vibration (mechanical and electric)