

# Freight Flow Consolidation in Presence of Time Windows

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**Abstract.** This contribution addresses the consideration of time windows in the optimization of multi-commodity network flows. For each node, one interval is specified in which the visitation is allowed. Applications in freight flow consolidation let this problem become interesting. An optimization model is proposed and a construction heuristic is presented. For improving the generated solutions, a genetic algorithm framework including several hill climbing procedures for local optimization, is configured.

## 1 Introduction

Multi-Commodity Network Flow Problems (MCBFP) are subject of a large number of scientific investigations. They are often consulted if a least cost flow of physical goods through a given transport network is searched. Typically, the costs represent the consumption of resources like time, fuel or budgets.

In logistics, the transportation of goods is only one particular step in the value creating process of a product. This step has to be synchronized with previous and subsequent processing steps. Therefore, time windows are specified for each single transport task in order to ensure a temporal coordination of the processing steps.

This article is about the optimization of time window-constrained flow of goods. In Section 2 the problem is stated in detail. Section 3 describes the used construction heuristic, Section 4 contains the description of a memetic improvement algorithm. The results of several numerical experiments are presented and discussed in Section 5.

## 2 Multi-Commodity Flow with Time Windows

A logistics service provider (LSP) is responsible for the reliable fulfillment of  $N$  pickup and delivery requests. Each request  $r_i$  expresses the need for the movement of a commodity  $i$  with capacity  $c_i$ . It has to be picked up at location  $p_i$  within the time window  $T_i^{pick}$  and unloaded within the time window  $T_i^{delivery}$  at location  $q_i$ . Since the LSP does not own any vehicles it pays a forwarding company for the physical execution of the requests. The minimization of the execution costs for the complete portfolio is required.

**Literature.** MCNFPs are targeted in several contributions. Here, it is referred to the comprehensive article [1]. The most famous special case is the shortest path problem [2].

Applications of MCNFP related to the optimization of the flow of goods in a transport network are described in [3], termed Freight Optimization.

The consideration of time windows has been described only in the context of the single-commodity shortest path problem [4].

**Problem Statement.** The set  $\mathcal{V} := \{p_1, \dots, p_N, q_1, \dots, q_N\}$  of all involved locations and the set  $\mathcal{A} := \mathcal{V} \times \mathcal{V}$  of arcs between pairs of the involved locations lead to the graph  $\mathcal{G} = (\mathcal{V}, \mathcal{A}, \gamma, \tau)$  representing the network available to fulfill the transport demands. The function  $\gamma$  is defined on the set of arcs and assigns the travel distance  $\gamma_{ij}$  for processing from  $i$  to  $j$  to each arc  $(i, j)$  and the time for traversing  $(i, j)$  is  $\tau_{ij}$ .

Each loading activity and each unloading activity is represented by a triple  $a(o) := (o, t_o^{start}, t_o^{end})$ . At location  $o$ , the corresponding loading or unloading activity takes place. It starts at time  $t_o^{start}$  and is completed at time  $t_o^{end} := t_o^{start} + d_o$  where  $d_o$  refers to the dwell time associated with  $o$ . The earliest allowed starting time of the operation associated with  $o$  is denoted by  $t_o^{min}$  and the latest allowed finishing time is named by  $t_o^{max}$ .

The way of commodity  $i$  through the graph  $\mathcal{G}$  originating from  $o_1 := p_i$  and terminating in  $o_{N_i} := q_i$  is determined in the *origin/destination-path* (*o/d-path*)  $\mathcal{P}_i := (a(o_1), a(o_2), \dots, a(o_{N_i}))$  of commodity  $i$ .

Along an o/d-path for commodity  $i$ , the operations are scheduled recursively starting from the earliest allowed execution time of the corresponding pickup-operation  $a(o_1)$ . The following starting times for  $i = 1, \dots, N_i - 1$  are computed by  $t_{o_{i+1}}^{start} = \max\{t_{o_i}^{min}, t_{o_i}^{start} + \tau_{o_i, o_{i+1}}\}$  and the finishing times are calculated by  $t_{o_i}^{end} := t_{o_i}^{start} + d_{o_i}$ .

The o/d-path  $\mathcal{P}_i$  is feasible for commodity  $i$  if it satisfies the time window conditions for its associated pickup operation  $a(p_i)$  and its associated delivery operation  $a(q_i)$ .

A fee  $F^{ij}(c)$  has to be transferred to the cooperating forward company for the movement of a commodity with a given capacity  $c$  along the arc  $(i, j)$ .

Typically,  $F$  is degressive with respect to increasing capacity  $c$ , so that it is more profitable to move one large commodity with capacity  $\alpha c$  along  $(i, j)$  than moving  $\alpha$  commodities each with capacity  $c$  along this arc. Thus the consolidation of several commodities associated with several requests that are shipped along an arc  $(i, j)$  starting at time  $t$  into a shipment  $S^{ij}(t) \subseteq \{r_1, \dots, r_N\}$  is profitable in certain cases. It leads to a reduced amount to be paid as long as the saved amount of fees dominates the additional feeder and distribution costs.

The overall sum of fees to be paid for realizing the feasible o/d-paths  $\mathcal{P}_1, \dots, \mathcal{P}_N$  is calculated as follows. At first, all shipments  $S^{ij}(t)$  occurring in  $\mathcal{P}_1, \dots, \mathcal{P}_N$  are identified. Then, the capacities  $C(S^{ij}(t))$  of the shipments  $S^{ij}(t)$  are computed by summing up the capacities of the included com-