Minimizing Total Delay in Fixed-Time Controlled Traffic Networks

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Abstract. We present two different approaches to minimize total delay in signalized fixed-time controlled inner city traffic networks. Firstly, we develop a time discrete model where all calculations are done pathwise and vehicles move on "time trajectories" on their routes. Secondly, an idea by GARTNER, LITTLE, and GABBAY (GLG) is extended to a continuous, linkwise operating model using "Link Performance Functions" to determine delays. Both models are formulated as mixed-integer linear programs and are compared and evaluated by PTV AG's simulation tool VISSIM 3.70.

1 Introduction

Controlling the inner city traffic by a "good" setting of relevant traffic light parameters is an appropriate way of reducing congestions in networks and delays in general, respectively. We consider a scenario of dense but almost steady traffic, which appears for example in periods of morning peaks of rush hour traffic. In such cases often fixed-time controlling is used since vehicle actuated or adaptive controlling cannot accentuate their advantages, which lie in the ability of adjusting to different traffic situations. Before we start describing the two optimization models, we review some notation used in traffic engineering.

First of all, we consider networks where there is a *light-signal system* at each intersection, including various single *traffic lights* some of which are combined into so-called *signal groups*. Each of these signal groups controls traffic throughput of a different direction, for example inbound, outbound, or turning traffic. For each signal group there is a *signal timing plan* determining the beginning and ending of the green- and red phase, the so called *red-green split*. After a predetermined amount of time, the *cycle time*, patterns of red and green recur. The most important parameter is the so called *offset*, which describes how light-signal systems of different intersections, or their signal timing plans respectively, are set relative to a given zero-point (Sec. 2) or are set relative to each other (Sec. 3). For both approaches cycle time, red-green split, and the vehicles' *travel time* on the links are fixed and the offset acts as decision variable.

2 A discrete path-based approach

Since in this first approach all delay analysis is done pathwise, we refer to the model as *path-based model* and restrict ourselves in the following to a single path. The model's main characteristic is its discrete structure, see Fig. 1. The cycle time is divided into T trajectories. For example, let $T = 40$; then at the beginning of each route each trajectory carries 0.555 cars per unit of time in case of an assumed traffic volume of 1000 vehicles per hour and a cycle time of 80 seconds. Below, we identify the cycle time with the parameter T and establish a partition into one trajectory per second of cycle time, which will be $T = 80$ for all quoted examples.

For a further development of the model it is necessary to introduce the following parameters: the set of all paths in the network is denoted by $\mathcal{P}$ and the edge set[1] of a path $P \in \mathcal{P}$ is given by $E(P) = \{e_1^P, \cdots, e_{\alpha_P}^P\}$ where α_P is the number of edges of P. The set of intersections, i.e. the set of different traffic-signal systems, is denoted by $\mathcal{K}$, whereas $\mathcal{R}(K)$ is the set of the signal groups corresponding to intersection K. Canonically, the parameter $\tau(e)$ stands for the integral travel time needed to traverse edge $e \in E(P)$ of path $P \in \mathcal{P}$. We will formulate the problem as a MIP and use the following

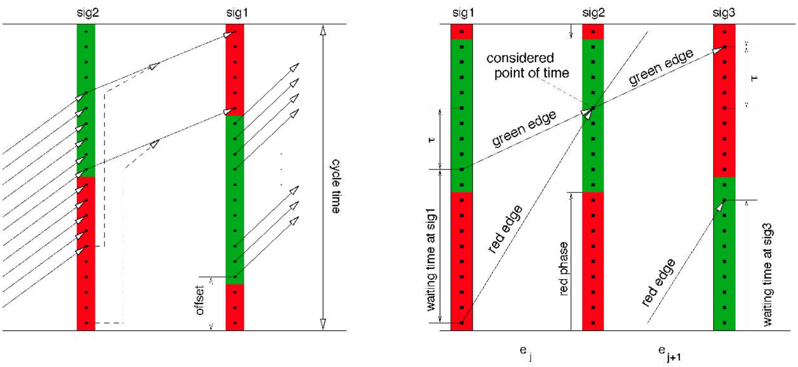

Fig. 1. Consecutive traffic signals of a particular path. At the beginning of each path each of the trajectories leads the same amount of vehicles.

variables: first, we have binary offset variables h, which are defined for each intersection and each point in time $\{1, \ldots, T\}$. For a $K \in \mathcal{K}$ only one variable $h[K, t]$ equals 1. The t_0 with $h[K, t_0] = 1$ corresponds to the offset value at intersection K. A second group of binary variables indicates whether a certain point of time $t \in \{1, \ldots, T\}$ at intersection $K \in \mathcal{K}$ and signal group $R \in \mathcal{R}(K)$ belongs to a green phase or not. These variables are denoted by

[1] We refer to links as edges although we do not actually use an underlying graph structure.