On Asymptotic Optimality of Permutation Schedules in Stochastic Flow Shops and Assembly Lines

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Abstract. For the stochastic flow shop problem with m machines, n jobs and operation processing times being independent identically distributed random variables, we consider permutation schedules — those in which each machine processes the jobs in the same order. For fixed m and increasing n, the asymptotic behavior of an arbitrary permutation schedule length is researched. Let T be the makespan of that schedule. Considering the maximum machine load L as a lower bound of the makespan, we show that for a wide class of operation processing time distributions the average-case ratio T/L converges to 1.

Thus, an arbitrary permutation flow shop schedule is asymptotically optimal with $n \to \infty$, and one can construct such schedule in a linear time. The same result is derived for the stochastic assembly line problem with m machines and n jobs.

1 Introduction and Definitions

Flow shop and assembly line are classical machine scheduling problems. In general case, both problems are $\mathcal{NP}$-hard, however several polynomial time algorithms are constructed in [3] (assembly lines) and [4], [5] (flow shops) producing solutions close to the optimum with absolute guarantee. It is interesting that in all algorithms the solutions were chosen within the class of permutation schedules. Thus, the idea to consider an arbitrary permutation schedule arose.

In the paper, we consider stochastic assembly line and flow shop problems and show that for an arbitrary permutation schedule the average case ratio converges to 1 asymptotically.

In general, each of the two scheduling problems considered below can be formulated as follows. Jobs $J_1, \ldots, J_n$ are to be processed on machines $M_1, \ldots, M_n$. Each job J_j consists of m operations $o_{1j}, \ldots, o_{mj}$. Operation o_{ij} is processed on machine M_i and requires time $p_{ij} \geq 0$ for its processing. Operation processing must satisfy the following conditions:

(B_1) *no simultaneous processing of any two operations on one machine is allowed;*

(B_2) *no operation preemption is allowed.*

Let s_{ij} denote the starting time of operation o_{ij} in a given schedule. The problem is to derive a schedule $S = \{s_{ij} \mid j = 1, \dots, n; \quad i = 1, \dots, m\}$ satisfying $(B_1) - (B_2)$ (and some additional requirements specific for each problem) and minimizing the maximum operation completion time (the makespan):

$$T(S) = \max_{j,i}(s_{ij} + p_{ij}). \tag{1}$$

Let p be the maximum operation length, L_i be the load of machine M_i and L be the maximum machine load:

$$p = \max_{i,j} p_{ij}, \quad L_i = \sum_{j=1}^{n} p_{ij}; \quad L = \max_i L_i. \tag{2}$$

Obviously, L is the lower bound on the length of any schedule S satisfying $(B_1) - (B_2)$. Precise formulations of the considered machine scheduling problems are given below.

In the Flow Shop Problem, there are two additional conditions on operation processing. Firstly,

(B_3) *no simultaneous processing of any two operations of one job is allowed*;

and secondly, each job must be processed on machines $M_1, \dots, M_m$ consequently:

(B_4) $s_{ij} \geq s_{i-1,j} + p_{i-1,j} \quad (i = 2, \dots, m;\ j = 1, \dots, n).$

Flow Shop Problem. *Derive a schedule satisfying* $(B_1) - (B_4)$ *and minimizing functional (1).*

In the assembly line problem, the last operation of each job must be processed on machine M_m:

(B_5) $s_{mj} \geq s_{ij} + p_{ij} \quad (i = 1, \dots, m-1;\ j = 1, \dots, n).$

Assembly Line Problem. *Derive a schedule satisfying* $(B_1), (B_2), (B_5)$ *and minimizing functional (1).*

2 Stochastic Machine Scheduling Problems

Formulating a stochastic machine scheduling problem, we assume all operation processing times p_{ij} being random variables with non-negative real values.

To solve a stochastic machine scheduling problem means to construct an algorithm producing a schedule S, the length $T(S)$ of which almost always (with increasing number of jobs n) satisfies the following relation for any $\varepsilon > 0$:

$$\lim_{n \to \infty} \mathbb{P}\left(\frac{T(S) - OPT}{OPT} < \varepsilon\right) = 1. \tag{3}$$

Schedule S is *asymptotically optimal* if it satisfies (3).