A Single Processor Scheduling Problem with a Common Due Window Assignment

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Abstract. We consider a single processor scheduling problem with a common due window assignment. Jobs completed within the due window incur no penalty, while other jobs incur either earliness or tardiness penalties. Boundaries of the due window are decision variables. The objective is to minimize the sum of the total weighted earliness, the total weighted tardiness and due window width penalty. This problem is an extension of the classical Weighted Earliness and Tardiness problem (**WET**). We proved that our problem is NP-hard and presented some properties of an optimal solution. To solve the problem we constructed a dynamic programming algorithm and a fully polynomial time approximation scheme. We also presented a polynomial time algorithm for the case with unit job processing times.

1 Introduction

In this paper we consider a single processor scheduling problem with a common due window assignment. Jobs completed within the due window incur no penalty, while other jobs incur either earliness or tardiness penalties. This problem is an extension of the well known Weighted Earliness and Tardiness problem (**WET**) [5] [3].

Problems with a common due window assignment were studied in [4], [6] and [8]. For comprehensive review of scheduling problems with due date and due window assignment see Gordon *et al* [2].

The considered problem can be stated as follows. Consider a set of n jobs $\mathbf{J} = \{1, \ldots, n\}$ to be processed on a single processor, which can handle only one job at a time. Processor idle times are forbidden. Each job j is characterized by processing time p_j and weights α_j, β_j. Let C_j denote the completion time of job j. The objective is to find a sequence of jobs and a left and a right edge of the due window (denoted by q_1 and q_2, respectively) such that the criterion

$$\sum \alpha_j E_j + \sum \beta_j T_j + \gamma(q_2 - q_1)$$

is minimized, where $E_j = \max\{q_1 - C_j, 0\}$ is earliness of job j, $T_j = \max\{C_j - q_2, 0\}$ is tardiness of job j and γ is a due window width penalty. The formulated problem will be referred to as $\mathbf{P}$.

We shall also consider two special cases of this problem, namely:
P1 - the case with unit processing times ($p_j = 1$ for each $j \in \mathbf{J}$),
P2 - the case with agreeable weights ($p_i/\alpha_i \leq p_j/\alpha_j \Leftrightarrow p_i/\beta_i \leq p_j/\beta_j$ for each $i, j \in \mathbf{J}$).

The problems are applicable in many manufacturing systems, where the negotiation between the producer and the customer occurs. The negotiation concerns the delivery of the final products. The producer should deliver the products before some established moment and, on the other hand the customer will not receive the products before the other established moment. This results in due window which corresponds to the time frame during which the customer is most willing to take delivery of the products. Products manufactured too early has to be held in inventory until the customer is ready to receive them. This results in such costs as capital, insurance and deterioration costs. On the other hand products manufactured too late incur costs connected with late charges, express delivery, or loss sales.

The remaining part of this paper is organized as follows. In Section 2 we prove that **P2** and **P** are NP-hard and present some properties of an optimal solution of **P**. We show, in Section 3, that **P1** can be solved in polynomial time. In Section 4 we present dynamic programming algorithm solving optimally **P2**. A fully polynomial time approximation scheme for **P2** is described in Section 5. Finally, we summarize the obtained results in Section 6.

2 Preliminary Analysis

In this section we present some properties of an optimal solution of **P** and prove that **P2** and **P** are NP-hard.

Before we start our consideration, let us first introduce some useful notation:
$\mathbf{T} \triangleq \{j \in \mathbf{J} : C_j > q_2\}$ - a set of tardy jobs,
$\mathbf{E} \triangleq \{j \in \mathbf{J} : C_j \leq q_1\}$ - a set of early jobs,
$\mathbf{W} \triangleq \mathbf{J} \setminus (\mathbf{E} \cup \mathbf{T})$ - a set of due window jobs.

Property 1. There exists an optimal solution to **P** where some job completes at q_1.

Proof. Assume that we have an optimal solution where no job finishes at q_1. Let u be a job such that $S_u < q_1 < C_u$. A cost of decreasing q_1 by $\varepsilon \leq q_1 - S_u$ is equal to:

$$\begin{aligned}\Delta_1 = \textstyle\sum_{j \in \mathbf{E}} \alpha_j (q_1 - C_j) + \gamma(q_2 - q_1) - \sum_{j \in \mathbf{E}} \alpha_j ((q_1 - \varepsilon) - C_j) \\ -\gamma(q_2 - (q_1 - \varepsilon)) = \varepsilon(\textstyle\sum_{j \in \mathbf{E}} \alpha_j - \gamma).\end{aligned}$$

Similarly, a cost of increasing q_1 by $\varepsilon \leq C_u - q_1$ is equal to:

$$\Delta_2 = \varepsilon(\gamma - \sum_{j \in \mathbf{E}} \alpha_j) = -\Delta_1.$$