A Note on the Relationship between Strongly Convex Functions and Multiobjective Stochastic Programming Problems

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Abstract. We consider multiobjective optimization problems in which objective functions are in the form of mathematical expectation of functions depending on a random element and a constraints set can depend on a probability measure. An efficient points set characterizes the multiobjective problems very often instead of the solution set in one objective case. A stability of the efficient points set (w.r.t. a probability measures space) and empirical estimates have been already investigated in the case when all objective functions were assumed to be strongly convex. The aim of the contribution is to present a modified assertions under rather weaker assumptions.

1 Introduction

Multiobjective optimization problems in which objective functions are in the form of mathematical expectation of functions depending on a random element and a constraints set can depend on the probability measure correspond to many economic activities (see e.g. [8]). Namely very often economic and social phenomena are influenced by random factors and, simultaneously, it is reasonable to evaluate them with several objective functions. We introduce the above mentioned problem in the following form.

Find

$$\inf \mathsf{E}_{F^\xi} g_i(x, \xi), \quad i = 1, 2, \ldots, l \quad \text{subject to } x \in \mathcal{K}_{F^\xi} \tag{1}$$

where g_i, $i = 1, \ldots, l$ are functions defined on $R^n \times R^s$, ξ is an s–dimensional random vector, F^ξ and P_{F^ξ} denote the distribution function and the probability measure of ξ; $\mathcal{K}_{F^\xi} \subset R^n$ is a nonempty set that generally can depend on F^ξ. (R^n, $n \geq 1$ denotes n–dimensional Euclidean space.)

Different approaches to the multiobjective problems have been introduced in the literature (see e.g. [2], [4], [15]). A vector of the objective functions is there often replaced by one "suitable" objective function (see e.g. the Markowitz model for a portfolio selection). However, evidently, an efficient

points set is an essential characterization of the multiobjective problem everywhere.

A complete information on the probability measure P_{F^ξ} is a necessary assumption to treat the problem (1). In applications this assumption is fulfilled very seldom, consequently, very often P_{F^ξ} has to be replaced by its approximation. However, to replace responsible P_{F^ξ} by some approximation it is necessary to deal with a stability (considered w.r.t. a probability measure space) and with corresponding statistical estimates. To this end a generalization of the Markowitz approach (known as the weight approach) has been employed. We define the corresponding problem by the relations.

Find

$$\inf\{\mathsf{E}_{F^\xi} g^\lambda(x, \xi) | x \in \mathcal{K}_{F^\xi}\}, \tag{2}$$

$$g^\lambda(x, z) = \sum_{i=1}^{l} \lambda_i g_i(x, z), \quad \lambda \in \Lambda, \quad x \in X, \quad z \in R^s,$$

$$\Lambda \quad = \{\lambda \in R^l : \lambda = (\lambda_1, \dots, \lambda_l), \ \lambda_i \in \langle 0, 1\rangle, \ i = 1, \dots l; \ \sum_{i=1}^{l} \lambda_i = 1\}.$$

The problem (2) is one–objective (parametric) optimization problem. It follows from the theory of the deterministic multiobjective problems that (under additional assumptions) there exists a relationship between the (properly) efficient points of the problem (1) and a solutions set of the problem (2). Consequently, to obtain assertions on the stability and empirical estimates concerning the problem (1) the results achieved for one–objective problems can be employed. To recall suitable results for one–objective case we can mention e.g. the papers [3], [6], [7], [14], [16]. However, to obtain stability results concerning the problem (1), an assumption on strongly convexity of the functions g_i, $i = 1, \dots, l$ has been supposed (see e.g. [10]). The aim of this contribution is to introduce a modified assertions under rather weaker assumptions.

2 Some Definitions and Auxiliary Assertions

A multiobjective deterministic optimization problem can be introduced as the problem.

Find

$$\min f_i(x), \ i = 1, \dots, l \quad \text{subject to } x \in \mathcal{K}. \tag{3}$$

f_i, $i = 1, \dots, l$ are functions defined on R^n, $\mathcal{K} \subset R^n$ is a nonempty set.

Definition 1. The vector x^* is an efficient solution of the problem (3) if and only if there exists no $x \in \mathcal{K}$ such that $f_i(x) \leq f_i(x^*)$ for $i = 1, \dots, l$ and such that for at least one i_0 one has $f_{i_0}(x) < f_{i_0}(x^*)$.