Generalized DEA-Range Adjusted Measurement

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Abstract. Data Envelopment Analysis (DEA) is designed to measure the efficiency of decision making units (DMUs). A scalarizing function reduces all numerical information on inputs and outputs of a DMU to a single efficiency score. A range adjusted measure normalizes input and output values through equalization factors, already known from Multicriteria Decision Making. Cooper et al (1999) introduce a range adjusted DEA model for a non-radial measure with variable returns to scale. We extend this approach by the help of a general DEA model framework. Calculation of suitable range adjusted measures is incorporated in a web-based DEA-tool. This tool allows processing of individual sets of data with a broad choice of model characteristics.

1 Introduction

Data Envelopment Analysis (DEA) is an approach to compare relative efficiency of decision making units (DMUs) such as schools, hospitals, libraries, banks etc. As the investigated DMUs use multiple inputs to produce multiple outputs direct comparisons are generally difficult. DEA makes use of linear programs to determine their relative performance which is defined as the distance between the efficient frontier and the DMU under consideration. Since the famous publication by Charnes, Cooper and Rhodes (CCR) [3] a large number of DEA models with various extensions and applications has been published [7,14].

Assumptions made for those models differ fundamentally with respect to technologies and aggregation rules of scalarizing functions. In contrast to the classical CCR model a range adjusted measure (RAM) utilizes technology-dependent ranges of feasible inputs and outputs [1,13]. Cooper, Park and Pastor [5] introduce a particular RAM model for a technology with variable returns to scale. Steinmann and Zweifel [11] discuss RAM properties for technologies with constant returns to scale. We extend the range adjusted measure by the help of a DEA model framework and illustrate the calculation of equalization factors in general.

The paper is organized as follows: section 2 introduces a DEA model framework. On this basis we analyze range adjusted models in section 3. Section 4 presents a brief description of the technical environment used for

calculations. A numerical example serves to illustrate the concept for various technologies and scalarizing functions.

2 DEA-Model Framework

In the following we assume a set of J DMUs. Each DMU$_j$ is characterized by M inputs x_{mj} $(m = 1, \ldots, M)$ and N outputs y_{nj}. The set of all feasible activities (productions) consists of linear combinations of all DMUs, incorporating activity levels $(\lambda_1, \ldots, \lambda_J)' \in \Lambda$:

$$\Lambda = \left\{ \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_J \end{pmatrix} \in \mathbb{R}_+^J \;\middle|\; \begin{array}{c} \underline{\lambda} \leqq \sum_j \lambda_j \leqq \overline{\lambda} \\ \ldots \end{array} \right\}.$$

Usually the sum of all activity levels has a lower bound $\underline{\lambda}$ and an upper bound $\overline{\lambda}$ $(0 \leqq \underline{\lambda} \leqq \overline{\lambda})$. As the dots indicate, additional conditions are conceivable: the multipliers λ_j are defined as integer variables, a restricted number of DMUs serving as a reference, etc. [7].

All efficient input-output combinations constitute the efficient frontier. A combination is called efficient, if no other one exists which is better in at least one input or output and not worse in the others. In the sense of Multicriteria Decision Making we minimize inputs and maximize outputs simultaneously.

The efficiency of a DMU$_0$ is calculated by comparing its inputs x_{m0} and outputs y_{n0} to a point on the efficient frontier. Deviations are captured by nonnegative variables d_m^- and d_n^+ respectively. Those are well known from goal programming [12,10]. Here a scalarizing function ψ aggregates all weighted input-output deviations [8,9]:

$$\begin{array}{lll} (DEAM) & \max & \psi(w_1^- d_1^-, \ldots, w_M^- d_M^-, w_1^+ d_1^+, \ldots, w_N^+ d_N^+) \\ & s.t. & 1)\ \sum_j x_{mj}\,\lambda_j + d_m^- = x_{m0} \qquad (m = 1, \ldots, M) \\ & & 2)\ \sum_j y_{nj}\,\lambda_j - d_n^+ = y_{n0} \qquad (n = 1, \ldots, N) \\ & & 3)\ \lambda \in \Lambda \\ & & 4)\ d_m^-, d_n^+ \geqq 0 \quad (m = 1, \ldots, M; n = 1, \ldots, N) \end{array}$$

In $(DEAM)$ we maximize the distance from any given DMU$_0$ to an efficient input-output-combination. If DMU$_0$ lies on the efficient frontier all deviations are zero and $\psi = 0$. Inefficient DMUs in contrast are characterized by a strictly positive efficiency score. The individual optimal value of scalarizing function ψ serves to rank inefficient DMUs. Many DEA models transform the objective function, so that efficient DMUs are indicated by $\psi = 1$.

The general framework $(DEAM)$ allows us to analyze various models developed in DEA. The models differ in

- technology (vrs, crs, nirs, ndrs, fdh, ...),