A fuzzy DEA approach

-Ranking production units in Taiwanese semiconductor industry-

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Abstract. The theoretical concept of data envelopment analysis (DEA) was published by Charnes, Cooper and Rhodes in 1978. It is a powerful tool for assessing the efficiency of decision making units (DMUs) by comparing their respective input-factors and output quantities. More precisely, the efficiency of a DMU is measured as the relation between the sum of weighted outputs and of weighted inputs, where the weights for a DMU are the solution of a linear optimization problem. But this approach suffers from a lack of objectivity due to the fact that every DMU wants an advantageous evaluation for itself, called self-appraisal in the respective literature. The purpose of this paper is to present an extension of the evaluation process in such a way that every DMU's performance will be calculated in an objective and fair manner. For this a fuzzy oriented approach will be formulated which yields a nonlinear optimization problem. The resulting input/output weights are a compromise of all DMU's individual interests. The so far developed theoretical concept will be applied to a company of Taiwanese semiconductor industry. Five DMUs' efficiencies will be calculated over three years. Professional software solves the nonlinear program and the results permit meaningful economical interpretations.

1 Introduction

Data envelopment analysis (DEA) is a mathematical technique used to calculate the efficiency of decision making units (DMUs), which can be organizations such as business firms, hospitals, stores, restaurants, etc. The results from the analysis yield information about the input-output relation for each DMU. The approach developed by Charnes et. al. [2] generalizes the single input output technical efficiency measure by Farrell [6] to a multiple input output technique. DEA is a powerful tool which is used for different fields of application [1], [5], [8]. In this paper we present a modification of the classical DEA technique which yields a multiple objective programming approach and make it a fair efficiency assessment. Therefore we pick up an idea by Chiang, Tzeng [3] and illustrate both, the philosophy of the classical DEA concept and its modification to a fuzzy DEA approach by a a real case study of Taiwanese semiconductor industry. Either model will be solved by a professional

software tool and we give interesting interpretations for the different results between both approaches. The paper is structured as follows.
First, in section 2, we sketch the mathematical model based on Charnes et. al. In section 3 we pick up the idea of a fuzzy oriented model which allows to evaluate DMUs in a more objective way than the classical DEA approach does. Section 4 applies the theoretical ideas to a company of Taiwanese semiconductor industry and compares the results between both approaches. A conclusion in section 5 completes this paper.

2 The classical DEA-method "CCR"

This section deals with the model which was initially proposed by Charnes, Cooper and Rhodes (CCR) [2]. We give a short survey of the mathematical background and avoid details. The reader finds a good discussion about the theory in [2], [4]. Efficiency or productivity analysis are managerial control tools for assessing the degree to which inputs are utilized in the process of obtaining outputs. If the production function is known the efficient input/output combinations can be determined directly. But it is very difficult to formulate an explicit functional relationship between inputs and outputs, in general. If the production function is not known, efficiency has to be estimated based on observed input and output data. Therefore the DEA tries to overcome the difficulties in such a way that CCR refers to the unknown production function as an envelope built up relative to observational data from all of the DMUs j, $j = 1, ..., n$. Let $\mathbf{x}_j = (x_{1j}, \cdots, x_{mj})$ be observed input factors and $\mathbf{y}_j = (y_{1j}, \cdots, y_{sj})$ be observed output factors of all DMUs $j = 1, ..., n$. Than the efficiency eff_{j_0} of a DMU j_0 is measured by the output to input ratio

$$\mathit{eff}_{j_0} = \frac{\sum_{r=1}^{s} u_r y_{rj_0}}{\sum_{i=1}^{m} v_i x_{ij_0}}, \tag{1}$$

where u_r and v_i represent virtual weights of the outputs and inputs, respectively. Thus, the problem of determing the efficiency of a DMU $j_0 \in \{1, ..., n\}$ becomes the problem of calculating the weights u_r, v_i in such a way that they solve the following fractional program

$$\begin{aligned} & \max \mathit{eff}_{j_0} = \frac{\sum_{r=1}^{s} u_r y_{rj_0}}{\sum_{i=1}^{m} v_i x_{ij_0}}, \; j_0 \in \{1, ..., n\} \\ s.t. \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, j = 1, ..., n \\ & u_r, v_i \geq 0, i = 1, ..., m, r = 1, ..., s. \end{aligned} \tag{2}$$

x_{ij} is the observed amount of the i-th input for the j-th DMU and y_{rj} is the observed amount of the r-th output for the j-th DMU. (2) for each DMU j_0 determines the maximal ratio of the sum of its weighted outputs divided by the sum of its weighted inputs. Charnes et al. postulate properties for the