A square law for power of positions in a network[1]

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Abstract. We present a model for the pressure between positions in a network. This notion is made operational through identifying the locations and sources of network value. Precisely these 'markets', where network value is obtained and generated, can be at stake and determine the pressure a position may experience. Using a limit approach, we also derive our square law of network power.

1. Introduction

We present a model for the notion of structural tension between pairs of positions in a value generating network. In a network, the value or worth of a position cannot be determined autonomously or exogenously. A position with a large number of links may derive more value from the network than a peripheral position can ever claim. Not only does this network value depend on the number of relations to other positions, it also depends on the network values of these positions: if a position somehow succeeds in gaining extra value, this also adds to the values of neighboring positions. Generally speaking, network value is the advantage positions have because of their connections within the network.

We assume that each pair of positions is aware of this mutual dependence, also through changes in network values of their common neighbors. As they 'feel' the effects of each other's moves, they monitor the other's network situation and are prone to react. We propose measures for this latent potential to engage in rivalrous behavior. Our measures are derived from the notion of competitive pressure between firms operating in the same industry, offering similar products, see for example (Chen 1996). This competitive pressure that a focal firm experiences from a competitor is determined by two factors: the strategic importance of each of the product markets the focal firm shares with the competitor, and that competitor's market share in these markets. In our model, markets and market shares are replaced with positions and network values. We make the notion of pressure in a network operational through identifying the locations and sources of network value for each position. Precisely these 'markets', where network value is ob-

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tained and generated, can be at stake and determine the pressure a position may experience from other positions through their common neighbors. Pressures or threats that focal positions experience from competitors, and vice versa, are measures for the structural tension between pairs of positions in the network.

We show that network power of positions, which is defined as pressure that is experienced by other positions, does depend on network value. We also show that the overall pressure that a position experiences can be reduced by creating new links. For one specific pressure-map, it is even true that a position with twice as much network value as another position, has four times as much power: our square law of network power.

In Sect. 2, we determine the network value for each position in a network. Pressure-maps are explained and computed in Sect. 3. Finally, in Sect. 4, we introduce a square law for network power.

2. Network value

Values of positions are recursively related to the values of positions to which they are linked. To determine these network values, we have to solve this circular dependence of the value of a position on the values of adjacent positions. A well-known and often used method is the eigenvector approach: take the eigenvector of the adjacency matrix, corresponding to the principal (largest) eigenvalue. See (Bonacich 1972; Wasserman and Faust 1994; Laslier 1997; Monsuur and Storcken 2002, 2004).

Let $G = (V,E)$ be a undirected network, where V is a finite set of positions and E a set of links between pairs of positions. We assume that the network is connected, meaning that each pair of positions can be (indirectly) linked by a finite sequence of links from E. Let A be the adjacency matrix, defined by $A_{ij} = 1$ if positions i and j are linked, and $A_{ij} = 0$ if there is no (direct) link. We take $A_{ii} = 1$ for all i, meaning that we assume that each position is adjacent to itself. Let $\underline{v}$ be the normalized eigenvector of A corresponding to the principal eigenvalue λ: $\underline{v} = \lim_{n\to\infty}\left(\frac{A^n}{\underline{e}^t A^n \underline{e}}\right)\underline{e}$. The components of $\underline{v}$ sum to one. Any nonnegative eigenvector of A is a multiple of $\underline{v}$. Then we let the network value c_i of a position i be its component of this vector $\underline{v}$. This means that the network value of a position is proportional to the sum of network values of adjacent positions.

For the graph of Fig. 1, the normalized eigenvector of the corresponding adjacency matrix is given by
[.0312, .0752, .0312, .119, .0939, .1074, .165, .1178, .0684, .1043, .0433, .0433] with eigenvalue 3.411238. The sum of all network values equals 1. For example, $c_g = 0.165$, meaning that 16.5% of the total network value generated by the (struc-