# Approximate Policies for Hybrid Production and Rework Systems with Stochastic Demand and Yield

Christian Gotzel, Karl Inderfurth

Otto von Guericke University of Magdeburg, Faculty of Economics and Management, P.O. Box 4120, 39106 Magdeburg, Germany

**Abstract.** We consider a production inventory problem under periodic review with stochastic demand and stochastically proportional yield. Defective products (i.e. the yield loss) arising from the unreliable production process can be temporarily stored in an inventory and passed through a rework process. Rework is completely reliable and brings defective products up to the same quality level as new ones, so that they can be used for demand fulfilment. We focus on the application of linear heuristic policies similar to those emerging from MRP application, and develop simple expressions for the computation of policy parameters. Results of a numerical study indicate that the heuristic approach performs quite well.

## 1 Introduction

The production-and-rework problem is common to several industrial production processes where a production reject arises as a result of deficiencies in the process or due to inconsistent raw material quality. Whereas a fraction of the production output can be directly used to fulfil the demand, the remaining fraction is passed through a rework process to receive products offering the desired quality level.

We consider a production inventory problem with stochastic yield and demand. Production yield is a random fraction of the production order. This yield model is usually referred to as stochastically proportional yield. Defective products, i.e., the yield loss, are stored in an inventory to be available for future rework. There is no disposal option. Yield and demand distributions are assumed to be stationary and independent. Procurement, rework, holding and shortage costs are strictly proportional. Any unsatisfied demands are backlogged. Production and rework leadtimes are deterministic but may differ.

Production-inventory problems with stochastic yield have been addressed by several authors. In [2] and [3] it is shown that the optimal control policy is rather complex even without a rework option. Applying linear heuristics for this prob-

lem as proposed in [1] and [3] has proven to give a good approximation. However, there is only few research directed to production systems with rework, mostly considering lot-sizing problems without regarding uncertainty, see [6] for an overview. An adjusted MRP approach for a hybrid production and remanufacturing planning is presented in [4].

In the next section of our paper we formulate a simple control policy for the problem under general leadtime conditions. In a subsequent section, we derive expressions for the determination of policy parameters and present results of a numerical study. We end with some final conclusions.

## 2 Control Policy

In the sequel we will consider a production and rework facility as described above. Events and decisions are timed in the following manner. At the beginning of period $t$, inventory levels are observed and the decision on the production quantity $p_t$ is made. In case of a zero production leadtime $\lambda_P$, the yield fraction $z_t \cdot p_t$ becomes immediately available at the serviceables inventory, otherwise it becomes available at the end of period $t+\lambda_P-1$. At the same time, the yield loss $(1-z_t)\cdot p_t$ enters the recoverables inventory. Then the decision on the rework quantity is made based on all information available at this time. Zero leadtime rework orders increase the serviceables inventory immediately whereas for rework leadtimes $\lambda_R>0$ the reworked items become available at the end of period $t+\lambda_R-1$. After this decision, the demand of period $t$ is realised and fulfilled from the inventory.

We use the following notation:

$h_S$ : serviceables holding cost per unit
$h_R$ : recoverables holding cost per unit
$v$ : shortage cost per unit
$d$ : stochastic demand with mean $\mu_d$ and variance $\sigma_d^2$
$z$ : stochastic yield fraction with mean $\mu_z$ and variance $\sigma_z^2$
$x_S$ : initial serviceables net inventory (on hand minus backorder)
$x_R$ : initial recoverables inventory
$p$ : production quantity
$r$ : rework quantity

Production and rework quantities are determined using a two-parameter $(S,M)$-policy with produce-up-to level $S$ and rework-up-to level $M$. This policy structure has proven a good performance in a hybrid production-remanufacturing system with external returns, see [5]. The production decision is based on a comparison