

Opening Structures

This chapter explores impacts of missing an axiom or a condition of an agreeable e.p.r.s structure. One may have in mind an intention to relax some of the axioms that ensure biagreemental or an entrepreneurial structure of an enterprise, and then ask for some other elements that are to compensate for them. Thus, some conditions are to be imposed that shape circulation of e.p.r.s among agents, and their arrangements of e.p.r.s, in the way that consistency of an e.p.r.s institution is ensured and e.p.r.s relations are under control. In particular, the conditions concerning coexpansion of e.p.r.s of copartner are relaxed so that an agreeable structure holds only up to some elements. The most important consequences have been already noted on simple e.p.r.s institutions, as discussed in Chapter 2 within 2.2.1, or enterprises on natural resources. This class of simple enterprises is modified to a class of enterprises that could be considered as simple only up to conjugation by an opening structure carrying some new e.p.r.s relation among partners. Namely, copartners within the class could accept simplified e.p.r.s rationality, but only up to conjugation by an opening structure obeying some new e.p.r.s conditions. A traditionally trained economist may have in mind market opening, although concept of an e.p.r.s opening includes innovation, initiation, R&D types of opening of economic institutions and similar. In the case of market opening, or simple market, one usually thinks of an economic device of measuring efficiency, and as a mechanism for welfare restructuring and redistribution. Note that a concept of an e.p.r.s opening also includes arbitrations and other forms of mediating devices between partners. From point of view of agreeable structure of such opening enterprises, they are truly different from the enterprises based on the established e.p.r.s rules or the simple *R&D* enterprises, as defined and described in the previous Chapter 2. Nevertheless, the e.p.r.s structural properties of these open institutions are so close to entrepreneurial conditions that all the familiar results for the enterprises based on e.p.r.s rules and simple e.p.r.s growth enterprises tend to have analogies here also. The enterprises which copartners' e.p.r.s arrangements are shaped in this way are called open enterprises. The fact that properties of these institutions are

so close to those based on simple e.p.r.s rules and simple growing enterprises, also allow us to consider them as institutions based on e.p.r.s rules or e.p.r.s enterprises. The examples given in this Chapter are indeed modifications of familiar simple enterprises and simple growing ones, but one should not think that these forms are the only examples. One may note that there are plenty of open enterprises that are not based on some fixed e.p.r.s rule or simple growing process at all.

In this Chapter basic definitions and properties behind the class of open enterprises are provided. In developing appropriate technic within mathematical economics on these issues, we continue within the abstract setting of general algebraic formalization of the economic phenomena. Here, we again distinguish e.p.r.s structures for simple open enterprises from those that incorporate more complex e.p.r.s structures of their openings.

3.1 Simple Opening

3.1.1 Definition and Main Properties

Recall from Section 2.2.1 in Chapter 2, when we considered properties of enterprises formed as simple ones, i.e., as enterprises on natural resources, that coexpansion of e.p.r.s has the property of being unaffected by any transposition map. Precisely, one has $\tau \circ \Delta = \Delta$, where τ is transposition map. This can be weakened requiring an enterprise to carry modified form of coexpansion of e.p.r.s, becoming only cocommutative up to conjugation by an element of $\mathcal{R} \in H \otimes H$, which obeys some further properties. This element \mathcal{R} is constituted as a quasitriangular e.p.r.s structure, and in the EPRT interpretation it carries a collection of e.p.r.s due to opening of a biagreement or simple enterprise H . Thus,

Definition 3.1 (Simple opening). *An open biagreement or a simple open enterprise is a pair (H, \mathcal{R}) , where H is a biagreement or a simple enterprise and $\mathcal{R} \in H \otimes H$, is an e.p.r.s transaction, that has its inverse, \mathcal{R}^{-1} , and obeys the following axioms of an opening,*

$$(\Delta \otimes id)\mathcal{R} = \mathcal{R}_{13}\mathcal{R}_{23}, \quad (id \otimes \Delta)\mathcal{R} = \mathcal{R}_{13}\mathcal{R}_{12} \quad (3.1)$$

$$\tau \circ \Delta h = \mathcal{R}(\Delta h)\mathcal{R}^{-1}, \quad \forall h \in H. \quad (3.2)$$

Writing $\mathcal{R} = \sum \mathcal{R}^{(1)}\mathcal{R}^{(2)}$, the notation used is

$$\mathcal{R}_{ij} = \sum 1 \otimes \dots \otimes \mathcal{R}^{(1)} \otimes \dots \otimes \mathcal{R}^{(2)} \otimes \dots \otimes 1,$$

the element of $H \otimes H \otimes \dots \otimes H$ which is \mathcal{R} in the i^{th} and j^{th} factors. Here, τ denotes the transposition map, and \mathcal{R} is called simple opening.