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Models and Basic Concepts

In this chapter we introduce three basic project scheduling problems: the time-constrained project scheduling problem, the resource-constrained project scheduling problem with renewable resources, and the resource-constrained project scheduling problem with cumulative resources. The time-constrained project scheduling problem consists in scheduling the activities of a project such that all temporal constraints are satisfied and some objective function is optimized. We review how temporal scheduling of the project can be performed by solving specific time-constrained project scheduling problems. We distinguish between two types of resources, namely renewable and cumulative resources, depending on whether or not resource availability at a given point in time is affected by the complete past project evolution. For both types of resources we show how to cope with resource constraints by establishing precedence relationships among the activities from so-called forbidden sets, whose joint resource requirements exceed the resource availability.

1.1 Temporal Constraints

1.1.1 Time-Feasible Schedules

A project can be considered to be a set of interacting tasks requiring time and resources for their completion. The structural analysis of the project provides a decomposition of the tasks into a set V of activities and a set E of precedence relationships among them. Set V consists of n activities $i = 1, \dots, n$ to be scheduled and two auxiliary activities 0 and $n + 1$, representing the project beginning and the project termination, respectively. The precedence relationships can be represented as activity pairs (i, j) where $i \neq j$, saying that the start time of activity i affects the earliest start time of activity j. Thus, $E \subset V \times V$ is some irreflexive relation in set V. Note that this relation may not be asymmetric if there are two activities $i, j \in V$ which mutually influence their earliest start times. The time estimation associates a duration

$p_i \in \mathbb{Z}_{\geq 0}$ with each activity and a time lag $\delta_{ij} \in \mathbb{Z}$ with each pair $(i, j) \in E$. An activity $i \in V$ is referred to as *fictitious activity* or *event* if $p_i = 0$. Otherwise, we speak of a *real activity*. The project beginning and termination, the receipt of materials, or milestones are examples of events. V^a and V^e respectively denote the sets of real activities and events of the project. We assume that the real activities must not be interrupted once they have been begun. Let S_i denote the start time of activity i, which has to be determined when scheduling the project in the temporal scheduling and resource allocation steps. If i is a fictitious activity, S_i is also termed the occurrence time of event i. The time lags δ_{ij} give rise to the *temporal constraints*

$$S_j - S_i \geq \delta_{ij} \quad ((i,j) \in E) \tag{1.1}$$

If $(i, j) \in E$, activity j cannot be started earlier than δ_{ij} units of time after the start of activity i. A nonnegative value of δ_{ij} corresponds to a *minimum time lag* $d_{ij}^{min} = \delta_{ij} \geq 0$ between activities i and j, whereas a negative value of δ_{ij} can be viewed as a *maximum time lag* $d_{ji}^{max} = -\delta_{ij} > 0$ between activities j and i. If $d_{ij}^{min} = p_i$, inequality (1.1) is referred to as a *precedence constraint* between activities i and j. For what follows, we establish the following convention.

Remark 1.1. The project is started at time 0 and must be completed by a prescribed deadline $\bar{d}$, i.e., $S_0 = 0$ and $S_{n+1} \leq \bar{d}$. The deadline is represented as a maximum time lag $d_{0,n+1}^{max} = \bar{d}$ between the project beginning 0 and the project termination $n + 1$.

The temporal constraints (1.1) connect the *start* times of activities i and j. Since by assumption activities must not be interrupted when being in progress,

$$C_i := S_i + p_i$$

is the completion time of activity i. Thus, start-to-start, start-to-completion, completion-to-start, and completion-to-completion relationships among activities can easily be transformed into one another (cf. e.g., Bartusch et al. 1988).

Remark 1.2. Some constraints that occur frequently in practice can be modelled by minimum and maximum time lags between activities (see Neumann and Schwindt 1997):

(a) Release date r_i for the start of activity i (head of i): $d_{0i}^{min} = r_i$.
(b) Deadline $\bar{d}_i$ for the completion of activity $i \in V$: $d_{0i}^{max} = \bar{d}_i - p_i$.
(c) Quarantine time q_i after the completion of activity i (tail of i): $d_{i,n+1}^{min} = p_i + q_i$.
(d) Fixed start time t_i for activity i: $d_{0i}^{min} = d_{0i}^{max} = t_i$.
(e) Simultaneous start of activities i and j: $d_{ij}^{min} = d_{ij}^{max} = 0$.
(f) Simultaneous completion of activities i and j with $p_i \geq p_j$: $d_{ij}^{min} = d_{ij}^{max} = p_i - p_j$.