A Semi-Discretized Heat Transfer Model for Optimal Cooling of Steel Profiles

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Summary. Several generalized state-space models arising from a semi-discretization of a controlled heat transfer process for optimal cooling of steel profiles are presented. The model orders differ due to different levels of refinement applied to the computational mesh.

19.1 The Model Equations

We consider the problem of optimal cooling of steel profiles. This problem arises in a rolling mill when different steps in the production process require different temperatures of the raw material. To achieve a high production rate, economical interests suggest to reduce the temperature as fast as possible to the required level before entering the next production phase. At the same time, the cooling process, which is realized by spraying cooling fluids on the surface, has to be controlled so that material properties, such as durability or porosity, achieve given quality standards. Large gradients in the temperature distributions of the steel profile may lead to unwanted deformations, brittleness, loss of rigidity, and other undesirable material properties. It is therefore the engineers goal to have a preferably even temperature distribution. For a picture of a such cooling plant see Figure 19.1.

The scientific challenge here is to give the engineers a tool to pre-calculate different control laws yielding different temperature distributions in order to decide which cooling strategy to choose.

We can only briefly introduce the model here; for details we refer to [Saa03] or [BS04]. We assume an infinitely long steel profile so that we may restrict ourselves to a 2D model. Exploiting the symmetry of the workpiece, the computational domain $\Omega \subset \mathbb{R}^2$ is chosen as the half of a cross section of the rail profile. The heat distribution is modeled by the instationary linear heat equation on $\Omega$:
\[ c \rho \partial_t x(t, \xi) - \lambda \Delta x(t, \xi) = 0 \quad \text{in } \mathbb{R}_{>0} \times \Omega, \]
\[ x(0, \xi) = x_0(\xi) \quad \text{in } \Omega, \]
\[ \lambda \partial_{\nu} x(t, \xi) = g_i \quad \text{on } \mathbb{R}_{>0} \times \Gamma_i, \partial \Omega = \bigcup_i \Gamma_i, \] (19.1)

where \( x \) is the temperature distribution \( (x \in H^1([0, \infty], X) \) with \( X := H^1(\Omega) \) being the state space), \( c \) the specific heat capacity, \( \lambda \) the heat conductivity and \( \rho \) the density of the rail profile. We split the boundary into several parts \( \Gamma_i \) on which we have different boundary functions \( g_i \), allowing us to vary the controls on different parts of the surface. By \( \nu \) we denote the outer normal of the boundary.

![Initial mesh, partitioning of the boundary, and a picture of a cooling plant.](image)

**Fig. 19.1.** Initial mesh, partitioning of the boundary, and a picture of a cooling plant.

We want to establish the control by a feedback law, i.e., we define the boundary functions \( g_i \) to be functions of the state \( x \) and the control \( u_i \), where \( (u_i)_i := u = Fy \) for a linear operator \( F \) which is chosen such that the cost functional

\[ J(x_0, u) := \int_0^\infty (Qy, y)_Y + (Ru, u)_U \, dt, \quad \text{with } y = Cx \] (19.2)

is minimized. Here, \( Q \) and \( R \) are linear selfadjoint operators on the output space \( Y \) and the control space \( U \) with \( Q \geq 0, R > 0 \), and \( C \in \mathcal{L}(X, Y) \).

The variational formulation of (19.1) with \( g_i(t, \xi) = q_i(u_i - x(\xi, t)) \) leads to:

\[ (\partial_t x, v) = -\int_\Omega \alpha \nabla x \nabla v \, dx + \sum_k \left( q_k u_k \int_{\Gamma_k} \frac{1}{c_k} v \, d\sigma - \int_{\Gamma_k} q_k x v \, d\sigma \right) \] (19.3)

for all \( v \in \mathcal{C}_0^\infty(\Omega) \). Here the \( u_k \) are the exterior (cooling fluid) temperatures used as the controls, \( q_k \) are constant heat transfer coefficients (i.e. parameters