An Evolutionary Algorithm for the Unconstrained Binary Quadratic Problems

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Abstract. In this paper a new evolutionary algorithm (EA) is described for the unconstrained Binary Quadratic Problem, which is to be used with small, medium and large scale problems as well. This method can be divided into two stages, where each stage is a steady-state EA. The first stage improves the quality of the initial population. The second stage uses concatenated, complex neighbourhood structures for the mutations and improves the quality of the solutions with a randomized k-opt local search procedure. The bit selection by mutation is based on an explicit collective memory (EC-memory) that is a modification of the flee-mutation operator (Sebag et al. 1997). We tested our algorithm on all the benchmark problems of the OR-Library. Comparing the results with other heuristic methods, we can conclude that our algorithm belongs to the best methods of this problem scope.

Keywords: Binary quadratic programming; Large-size problems; Evolutionary algorithm.

1 Introduction

The general formulation the unconstrained binary quadratic programming problem (BQP) is the following:

\[ \text{Max } f(x) = x^T Q x + c^T x \]

where \( x \in \{0, 1\}^n \), \( Q \in \mathbb{R}^{n \times n} \) is an \( n \times n \) symmetric rational matrix.

BQP has a central role in combinatorial optimization. A large number of problems can be formulated as maximization of quadratic real values function in 0-1 variables. For that reason, BQP has been referred to as “the mother of all the combinatorial problems” (Bjorndal et al. 1995). E.g. BQP is equivalent to many classical combinatorial optimization problems such as maximum cut, maximum clique, maximum vertex packing, and maximum vertex cover. As important application we can see e.g. machine scheduling, capital budgeting, financial analysis and molecular conformation problems.
The techniques which can be used to find the exact optimal solution are the branch and bound and branch and cut methods (e.g. (Helmberg and Rendl 1998), (Horst et al.2000), Pardalos and Rodgers 1990)). We can also use linearization techniques as well (e.g. the reformulation-linearization technique (Sherali and Adams 1998)), which converts nonlinear mixed-integer or zero-one problems into linear ones. This technique can be used not only to construct exact solution algorithms, but also to design powerful heuristic procedures.

Generally problems of sizes larger then n=100 cannot be solved in an acceptable time. Since most real-world problems are large-size problems, heuristics are used to find good solutions within a reasonable time (Glover et al. 2002). A large number of heuristic methods have been developed for solving the BQP. Various heuristic methods are also frequently used, such as one-pass heuristics (Glover et al. 2002), simulated annealing (SA) (Beasley 1999), (Katayama and Narihisa 2001), tabu search (TA) (Beasley 1999), (Glover et al. 1998), evolutionary algorithm (EA) and versions (e.g. genetic algorithm (GA), evolutionary strategy (ES)) (Lodi et al. 1999), (Merz and Freisleben 1999), scatter search (Glover 1997), memetic algorithms (MA) (Merz and Katayama 2001), as well as differently iterated search techniques (e. g. the parthenogenetic algorithms (PA) (Katayama and Narihisa 2001) or various subgradient-type methods (Shor 1998).

In this paper, we present a new heuristic method to solve the BQP. This heuristic is an EA that consists of 2 consequent stages. The first stage improves the quality of the initial population. The second stage uses concatenated, complex neighbourhood structures for the mutations, improves the quality of the solutions with a randomized $k$-opt local search procedure and uses a special filter and restart technique. The bit selection by mutation is based on an explicit collective memory (EC-memory) that is a modification of the fleemutation operator (Sebag et al. 1997). The efficacy of the method was studied on the OR-Library benchmarks: the small, medium and large scale problems were all successfully solved. Comparing the results to others methods, we can state that our algorithm belongs to the best heuristic’s methods of this problem scope.

In section 2, we describe our EA in general. Section 3 includes implementation details of our EAs. In Section 4, we present our computational experience, and we compare our results with other heuristic’s results. Section 5 contains concluding remarks.

2 The principle of the new evolutionary algorithm

2.1 The structure of the algorithm

Hybrid EAs are frequently used for solving combinatorial problems. These methods improve the quality of the descendent solution for example with the application of a local search procedure, SA, or TS. The constitution of these