Approximation of Fuzzy Functions by Extended Fuzzy Transforms

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Abstract. A fuzzy approximation method called fuzzy transforms for approximation of continuous function is presented in this paper. It is shown how can be fuzzy transforms naturally generalized for functions with more variables. A fuzzy function as an approximated mapping is considered. This leads to an extension of fuzzy transforms for fuzzy function as well as to an extension of generalized fuzzy transforms for fuzzy functions with more variables. It is shown how the proposed method can be used as so called learning to obtain a fuzzy rule base for fuzzy control.

Key words: Fuzzy sets, Approximation, Fuzzy approximation, Fuzzy transforms, Normal forms, Fuzzy control

1 Introduction

Fuzzy transforms (in short F-transforms) have been already several times introduced in a number of publications.Perfilieva I. presented this technique of approximate representation of continuous functions in [4], its application to numeric methods of integrations and solution of ordinary differential equations in [1, 2]. Another application has been published in [6].

The main idea consists in the replacement of an continuous function on a real closed interval by its discrete representation (using the direct F-transform). Afterwards, the discrete representation is transformed back to the space of continuous functions (using the inverse F-transform). The result, obtained by applying both F-transforms is a good simplified approximation of an original function.

In fuzzy control we work with imprecise data and a crisp function describing some proces is described by a fuzzy relation. And any fuzzy relation can be viewed as a fuzzy function. This leads to an idea to extend the method of F-transforms for fuzzy functions to be able to apply it in fuzzy control.
2 Fuzzy Transforms

This section is devoted to Fuzzy transforms - fuzzy approximation method, first published byPerfilieva I. and Chaldeeva E. in [4]. This technique belongs to the area called numerical methods on the basis of fuzzy approximation models. An interval \([a, b]\) of real numbers will be considered as a common domain of all functions in this section.

Definition 1. Let \(x_i = a + h(i - 1)\) be nodes on \([a, b]\) where \(h = (b - a)(n - 1)\), \(n \geq 2\) and \(i = 1, \ldots, n\). We say that functions \(A_1(x), \ldots, A_n(x)\) defined on \([a, b]\) are basic functions if each of them fulfills the following conditions:

- \(A_i : [a, b] \rightarrow [0, 1]\), \(A_i(x_i) = 1\),
- \(A_i(x) = 0\) if \(x \notin (x_{i-1}, x_{i+1})\) where \(x_0 = a, x_{n+1} = b\),
- \(A_i(x)\) is continuous,
- \(A_i(x)\) strictly increases on \([x_{i-1}, x_i]\) and strictly decreases on \([x_i, x_{i+1}]\),
- \(\sum_{i=1}^{n} A_i(x) = 1\), for all \(x\),
- \(A_i(x_i - x) = A_i(x_i + x)\), for all \(x \in [0, h]\), \(i = 2, \ldots, n - 1, n > 2\),
- \(A_{i+1}(x) = A_i(x - h)\), for all \(x, i = 2, \ldots, n - 2, n > 2\).

We can say that functions \(A_i(x)\) determine a fuzzy partition of real interval \([a, b]\).

The technique of fuzzy transforms is based on two transforms - the direct one and the inverse one. The direct fuzzy transform is a mapping which maps continuous functions on \([a, b]\) into the space of real vectors. The inverse F-transform maps a real vector back to the space of continuous functions. We repeat definitions given in [4].

Definition 2. 1. Let \(f(x)\) be arbitrary continuous function on \([a, b]\) and \(A_1, \ldots, A_n\) basic functions determining a fuzzy partition of \([a, b]\). We say that an \(n\)-tuple of real numbers \([F_1, \ldots, F_n]\) is the direct F-transform of \(f\) with respect to \(A_1, \ldots, A_n\) if

\[
F_i = \frac{\int_a^b f(x) A_i(x) \, dx}{\int_a^b A_i(x) \, dx}. \tag{1}
\]

2. Let \(f(x)\) be a function known at nodes \(x_1, \ldots, x_r \in [a, b]\) and \(A_1, \ldots, A_n\) basic functions determining a fuzzy partition of \([a, b]\). We say that an \(n\)-tuple of real numbers \([F_1, \ldots, F_n]\) is the direct F-transform of \(f\) with respect to \(A_1, \ldots, A_n\) if

\[
F_i = \frac{\int_a^b f(x) A_i(x) \, dx}{\int_a^b A_i(x) \, dx}. \tag{2}
\]