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An Introduction to Tensors

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Summary. This chapter is a short introduction into tensor fields, some basic tech-
niques from linear algebra, differential geometry and the mathematical concept of
tensor fields are presented. The main goal of this chapter is to give readers from dif-
ferent backgrounds some fundamentals to access the research papers in the following
chapters.

1.1 Some Linear Algebra

Remark: Since we are only able to sketch out some of the basic facts of linear
algebra, the reader is referred to a comprehensive body of literature on the
topic. For example, the book by Fuhrmann [1] provides an introduction in a
modern language.

1.1.1 Bases and Basis Transforms

Let $U$ a vector space over a field $\mathbb{F}$ (e.g. $\mathbb{F} = \mathbb{R}$ or $\mathbb{F} = \mathbb{C}$). A set of elements
$\{a_1, \ldots, a_n\} \subset U$, $n \in \mathbb{N}$, is called a basis if every $u \in U$ admits a unique
non-trivial linear combination of the $a_i$ over $\mathbb{F}$:

$$u = u_1 a_1 + \cdots + u_n a_n$$

The coefficients $u_i$ give rise to the vector notation of $u \in U$ with respect to
this basis as

$$u = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix}.$$  

We say that $U$ has dimension $n$, since every basis has exactly $n$ elements and
every $u \in U$ can be described by $n$ elements of $\mathbb{F}$ (coordinates).

Let $V$ a vector space over $\mathbb{F}$ with basis $(b_1, \ldots, b_m)$, $m \in \mathbb{N}$. Hence,
$\dim V = m$. A linear map $L : U \to V$ is a map satisfying