18 Modeling of Heat and Mass Transfer in SSF Bioreactors

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18.1 Introduction

Chapters 22 to 25 present case studies in which mathematical models are used to explore the design and operation of various SSF bioreactors. Chapters 18 to 20 address the basic principles of the balance/transport sub-models of these bioreactor models.

The various phenomena that need to be described by the balance/transfer sub-model, such as conductive and convective heat transfer, were covered in a qualitative manner in Chap. 4. The current chapter shows the mathematical expressions that are used to describe these phenomena. The aim is not to teach heat and mass transfer principles to a depth that will allow readers to construct the appropriate mathematical expressions themselves. Rather, it is to enable readers to inspect a mathematical model of an SSF bioreactor and recognize which transport phenomena are described by the model, on the basis of the various terms that appear within the model equations. These terms include various system, thermodynamic, and transport parameters. Chapters 19 and 20 quote some typical values that have been used for these parameters and give some general advice as to how they might be determined experimentally. However, please note that detailed experimental instructions are not provided.

18.2 General Forms of Balance Equations

The transport/balance part of a mathematical model of a bioreactor consists of mass and energy balance equations. Such an equation expresses how a key system variable changes over time and includes terms that describe various phenomena that affect that variable.

Regardless of what the units of the variable of interest are, the balance equation should initially be written in such a way that all of its terms have units of either kg h\(^{-1}\), in the case of a mass balance, or J h\(^{-1}\), in the case on an energy balance. After this the equation can be rearranged if necessary to isolate the variable of interest.
As an example, an energy balance will appear in the form:

\[
\frac{d}{dt} (m_{\text{bed}} C_{P_{\text{bed}}}) = \pm Q_A \pm Q_B \pm Q_C + \ldots + r_Q,
\]

where \(m_{\text{bed}}\) is the mass of the bed (kg), \(C_{P_{\text{bed}}}\) is the overall heat capacity of the bed (J kg\(^{-1}\) °C\(^{-1}\)), \(T_{\text{bed}}\) is the bed temperature (°C), \(r_Q\) is the rate of metabolic heat production (J h\(^{-1}\)) (see Eq. (17.1)), and \(Q_A\), \(Q_B\), and \(Q_C\) represent expressions that describe the rates at which different heat transport phenomena occur (all in J h\(^{-1}\)). Whether they are added or subtracted will depend on whether they tend to increase or decrease the energy of the bed. The current chapter addresses the question of how these various “\(Q\)-terms” can be written mathematically. Equation (18.1) says that the rate of change in the amount of energy stored within the bed (in J h\(^{-1}\)), which is represented by the left hand side of the equation, depends on the rates of the various processes that either add energy to the bed or remove energy from it. Equation (18.1) is written in terms of energy, because this is a conserved quantity, whereas temperature is not. Later on, this equation will be rearranged to leave only \(dT_{\text{bed}}/dt\) on the left hand side, since this is actually the system variable of interest.

The construction of the left hand side of Eq. (18.1) can be understood by assuming that initially a substrate bed is at a temperature \(T_{\text{initial}}\), and during the fermentation a part of the metabolic heat released by growth remains in the bed, increasing its temperature. The amount of “extra energy” held within the substrate bed due to this increase in temperature is given by the product of the mass of the bed, the heat capacity of the bed and the temperature difference:

\[
\text{"Extra Energy"} = m_{\text{bed}} C_{P_{\text{bed}}} (T_{\text{bed}} - T_{\text{initial}}),
\]

which can be shown by determining the units of the result of the calculation (i.e., kg × J kg\(^{-1}\) °C\(^{-1}\) × °C simplifies to give J).

On the other hand, a mass balance, for example, a balance on the water in the bed, will appear in the form:

\[
\frac{dM_{\text{water}}}{dt} = \pm R_A \pm R_B \pm R_C + \ldots + r_W,
\]

where \(M_{\text{water}}\) is the overall mass of water in the bed (kg), \(r_W\) is the rate of metabolic water production (kg h\(^{-1}\)) (See Eq. (17.2)), and \(R_A\), \(R_B\), and \(R_C\) represent the rates of various mass transfer phenomena that involve water (all in kg h\(^{-1}\)). Whether they are added or subtracted will depend on whether they tend to increase or decrease the amount of water in the bed. The current chapter addresses the question of how these various “\(R\)-terms” can be written mathematically. Equation (18.3) says that the rate of change in the mass of water in the bed (in kg h\(^{-1}\)), which is represented by the left hand side of the equation, depends on the rates of the various processes that either add water to the bed or remove water from it.

Note that it may be desirable to have an equation that expresses directly the rate of change of the water content of the bed (\(W\), kg-water kg-dry-solids\(^{-1}\)), and not the total mass of water in the bed. Even in this case, the equation should initially