Chaos, Nonlinearity, Complexity: A Unified Perspective

A. Sengupta

Department of Mechanical Engineering, Indian Institute of Technology Kanpur, Kanpur 208016, INDIA
E-Mail: osegu@iitk.ac.in

Summary. In this paper we employ the topological-multifunctional mathematical language and techniques of non-injective illposedness developed in [30] to formulate a notion of ChaNoXity — Chaos, Nonlinearity, Complexity — in describing the specifically nonlinear dynamical evolutionary processes of Nature. Non-bijective ill-posedness is the natural mode of expression for chanoxity that aims to focus on the nonlinear interactions generating dynamical evolution of real irreversible processes. The basic dynamics is considered to take place in a matter-negmatter (regulating matter, defined below) kitchen space $X \times \mathcal{X}$ of Nature that is inaccessible to both the matter ($X$) and negmatter ($\mathcal{X}$) components. These component spaces are distinguished by opposing evolutionary directional arrows and satisfy the defining property

$$(\forall A \subseteq X, \exists \mathcal{A} \subseteq \mathcal{X}) \text{ s.t. } (A \cup \mathcal{A} = \emptyset).$$

Dynamical equilibrium is considered to be represented by such competitively collaborating homeostatic states of the matter-negmatter constituents of Nature.

The reductionist approach to science today remains largely the dominant model. It fosters the detailed study of limited domains in individual subdisciplines within the vast tree of science. However, over the past 30 years or so, an alternative conceptual picture has emerged for the study of large areas of science which have been found to share many common conceptual features, regardless of the subdiscipline, be it physics, chemistry or biology. Self-organization and complexity are the watchwords for this new way of thinking about the collective behaviour of many basic but interacting units. In colloquial terms, we are talking about systems in which ‘the whole is greater than the sum of parts’.

Complexity is the study of the behaviour of large collection of such simple, interacting units, endowed with the potential to evolve with time. The complex phenomena that emerge from the dynamical behaviour of these interacting units are referred to as self-organizing. More technically, self-organization is the spontaneous emergence of non-equilibrium structural reorganizations on a macroscopic level, due to the collective interactions.
between a large number of (usually simple) microscopic objects. Such structural organizations may be of a spatial, temporal or spatio-temporal nature, and is thus an emergent property.

For self-organization to arise, a system needs to exhibit two properties: it must be both dissipative and nonlinear. Self-organization and complexity are essential scientific concepts for understanding integrated systems whether in physics, biology or engineering ··· with a much more ‘holistic’, yet equally rigorous, scientific perspective compared with the reductionist methods, and so provide new insights into many of the more intellectually challenging concepts, including the large-scale structure of the Universe, the origin and evolution of life on Earth (and more widely in the cosmos), consciousness, intelligence and language.

There is, therefore, a general and conceptual framework for the description of self-organizing phenomena, of a theoretical and essentially mathematical nature. This more or less boils down to the theory of nonlinear dissipative dynamical systems.

Coveney [8]

10.1 Introduction

A dissipative structure is an open, out-of-equilibrium, unstable system that maintains its form and structure by interacting with its environment through the exchange of energy, matter, and entropy, thereby inducing spontaneous evolutionary convergence to a complex, and possibly chaotic, equilibrated state. These systems maintain or increase their organization through exergy destruction in a locally reduced entropy state by increasing the entropy of the “global” environment of which they are a part. This paper applies the mathematical language and techniques of non-bijective, and in particular non-injective, ill-posedness and multifunctions introduced and developed in [30] to formulate an integrated approach to dissipative systems involving chaos, nonlinearity and complexity (ChaNoXity), where a complex system is understood to imply

- an assembly of many interdependent parts
- interacting with each other through competitive nonlinear collaboration
- leading to self-organized, emergent behaviour.¹

¹ Competitive collaboration — as opposed to reductionism — in the context of this characterization is to be understood as follows: The interdependent parts retain their individual identities, with each contributing to the whole in its own characteristic fashion within a framework of dynamically emerging global properties of the whole. A comparison with reductionism as summarized in Fig. 10.10c, shows that although the properties of the whole are generated by the parts, the individual units acting independently on their own cannot account for the emergent global behaviour of the total.