10. Indeterminacy in Discrete-Time Infinite-Horizon Models

Kazuo Nishimura
Kazuo Nishimura, Institute of Economic Research, Kyoto University, Japan

Alain Venditti
Alain Venditti, CNRS, GREQAM, Marseille, France

10.1 Introduction

Recently there has been an increasing interest in sunspot equilibria as a possible explanation of business cycle fluctuations. In a macroeconomic context, sunspot fluctuations is a topic that dates back to the early work of Shell [25], Azariadis [1] and Cass and Shell [10]. This renewed interest is explained by the fact that during the last decade a variety of economic models that incorporate some degree of market imperfections have been shown to exhibit multiple equilibria and local indeterminacy.[1] As shown by Woodford [29], the existence of sunspot equilibria is closely related to the indeterminacy of perfect foresight equilibrium.

Indeterminacy, or multiple equilibria, is known to occur in dynamic models with small market distortions and generates some coordination problems. Basically, the occurrence of indeterminacy needs a mechanism such that, starting from an equilibrium, if all agents were simultaneously to increase their investment in, say, the capital good, the rate of return on this good would tend to increase, and in turn set off relative price changes that would drive the economy back towards the steady state. In one-sector models, such a mechanism may be associated with external effects in production and increasing returns. However, in a two sector model, the rate of return and marginal product of capital depend not only on factor inputs, but also on the composition of output and thus on the relative factor intensities. An increase of the production and the stock of capital following an increase in its price may well increase its rate of return. Therefore constant aggregate returns at the social level are compatible with indeterminacy if there are minor external effects in some of the sectors.

[1] See Benhabib and Farmer [5] for an extensive bibliography.

In this chapter we will present the main conditions for the occurrence of indeterminacy in one and two-sector optimal growth models extended to include market imperfections based on technological external effects. We will focus almost exclusively on discrete-time models. We will distinguish between different formulations for externalities which will be in general associated with different assumptions concerning the returns to scale at the social level. Following Romer [24], one-sector models are characterized by global external effects coupled with increasing social returns. We will show that indeterminacy of equilibria is fundamentally based on the consideration of endogenous labor demand and externalities coming both from capital and labor. In two-sector models, Benhabib and Farmer [4] have introduced sector-specific external effects. While their initial formulation assumed increasing social returns, most of the papers that followed the contribution of Benhabib and Nishimura [7] are based on constant returns to scale at the social level. We will show that some simple conditions on capital intensity differences across sectors generate some amplification mechanisms that produce the existence of indeterminate equilibria.

The chapter is organized as follows. Section 2 presents one-sector models. Two-sector models with Cobb-Douglas technologies, complete depreciation of capital and sector-specific externalities are analyzed in Section 3. Section 4 is devoted to the presentation of similar two-sector models but with CES production functions. The cases with symmetric and asymmetric elasticities of capital-labor substitution are consecutively considered. In Section 5 we discuss extensions of the two-sector Cobb-Douglas formulation. Firstly, we present how the conditions for local indeterminacy are modified when partial depreciation of capital is assumed. Secondly, we introduce a formulation for intersectoral externalities that is compatible with both sector-specific and global externalities specifications. We will then show how additional intersectoral mechanisms provide new room for local indeterminacy. Finally, in Section 6, other formulations of infinite-horizon models are explored. We first deal with the consideration of aggregate models with capacity utilization in which the speed of capital depreciation is endogenously determined. Then we present two-sector models derived from general technologies.

10.2 One-Sector Models

One-sector discrete-time models with Romer-type [24] global externality and increasing returns at the social level have been considered initally by Kehoe [14] and Boldrin and Rustichini [9]. The aggregate production function is augmented to include a new factor which represents the effect of knowledge on production and productivity:

$$Y_t = F(K_t, L_t, A_t)$$

with A_t the externality at time t which will be equal at the equilibrium to K_t/L_t. For any given A, $F(.,.,A)$ is increasing, concave and homogeneous of