# 13. Equilibrium Dynamics with Many Agents

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## 13.1 Introduction: Ramsey's Steady State Conjecture

Frank Ramsey's seminal article [90] on optimal capital accumulation is now widely regarded as the foundation of macroeconomic dynamics.[1] The original model is cast as an optimal saving problem to be solved by an omniscient central planner acting over an infinite horizon to maximize discounted utility subject to the economy's resource constraints and given initial endowment of capital goods. Ramsey's planner operates in a deterministic world. Modern researchers have reinterpreted this model as one of intertemporal equilibrium.[2] An infinitely-lived representative consumer takes the place of Ramsey's central planner. This household is assumed to maximize lifetime discounted utility over the infinite horizon with perfect foresight regarding the time paths of all relevant prices. Equilibrium profiles satisfy a materials balance constraint at each time so that demand for goods equals their supply at each instant and in addition, the production sector's profits are maximized.[3]

Ramsey did not focus on an equilibrium interpretation of his optimal saving-accumulation framework. However, in the latter part of his paper he did formu-

[1] Ramsey begins his paper with a detailed analysis of an optimal saving-accumulation problem when the planner does **not** discount future utilities. He criticizes the idea of the **planner** discounting future utilities as one of a failure of imagination. However, he also articulates a fully developed optimal saving-accumulation model for the discounted case and it is this version of the theory that has proven so useful in modern macroeconomics. Modern advanced macro texts emphasizing the Ramsey model include Azariadis [3], Farmer [45], and Ljungqvist and Sargent [69].

Ramsey's contributions to mathematics and philosophy are found in [91]. A presentation and assessment of his work in those areas as well as economics can be read in [94]. Both references include biographies of Ramsey's short life.

[2] The connections between Ramsey's theory and alternative representations of intertemporal equilibrium for representative agent models are detailed in [8].

[3] It turns out in the specifications studied in this chapter that present value profits are maximized if and only if current value profits are maximized at each time.

late a model of stationary equilibrium — one with all variables constant over time. That model involved several types of infinitely lived households differentiated by their fixed rates of time preference. He conjectured that the model's solution would take the form of having the most patient consumer enjoying the largest sustainable consumption and in possession of the economy's capital stock while the remaining households consumed only at the minimum level necessary to sustain their lives.[4] This two-class solution suggested a very uneven distribution of consumption and wealth in a stationary state — a distribution entirely driven by the economy's fundamental taste and technology parameters.

Ramsey did not spell out the details of the equilibrium model or exactly what is meant by an equilibrium in his two-class theory. The purpose of this chapter will be to survey one interpretation of Ramsey's multi-agent model, solve for the steady state distribution and examine the models' dynamics within well-specified theories of intertemporal equilibrium.[5] The resulting analysis will show that there are fundamental differences between the dynamics of the representative agent model and one with heterogenous households. Not only will the long-run distribution of income and wealth differ from the representative agent outcome, but so will the dynamics. Indeed, the convergence of the economy to the long-run steady state from arbitrary initial conditions characteristic of Ramsey's optimal accumulation – representative agent equilibrium model will only hold for **some** specifications of preferences and technology in the multi-agent setup. Complicated dynamics at the aggregate level can arise even with very unequal income and wealth distributions evolving over time. The dynamic properties of the heterogenous agent story are thus richer than those of the representative agent model even when the aggregate economic variables tend over time to their long-run steady state values.

## 13.2 Impatience and the Distribution of Wealth

Ramsey's conjecture that with households having different rates of impatience, the steady state equilibrium would have very unequal income and wealth distributions was not a particularly new idea at the time his paper was published. The notion that time preference differences operating in a market economy might promote long-run differences in income and wealth can be found in the writings of such eminent economists as John Rae in 1834 [89] and in several books by Irving Fisher beginning with his great work on the rate of interest first published in 1907 [46]. This literature is reviewed next along with Stiglitz's [101] "descriptive" model of wealth distribution. His framework does not specify explicit maximizing behavior for the consumption-saving decisions undertaken

[4] Ramsey's savers could achieve a state of *bliss* either by holding the maximum possible capital stock — *capital saturation*, or by consuming at a level giving rise to *utility saturation*.

[5] This chapter reviews only discrete time models. It also omits extensions to international trade as found in [4], [104], and [107].