

# 14. Dynamic Games in Economics

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## 14.1 Introduction

This section provides a general idea of the contents and organization of this survey of a large body of research in economics and related fields loosely defined by the adoption of the common methodology of dynamic games. We note at the outset that this class of games has also been referred to in various contexts as stochastic games<sup>2</sup>, state-space games, sequential games, Markov games and difference (or differential) games. Given the breadth of this task and the long time span of the relevant strands of literature, some omission is inevitable.

The paradigm of dynamic games has long appeared natural and appealing in economic modeling, and has been adopted in many different subfields of the discipline. Various factors have prevented an even more widespread use of this theory, including in particular the complexity of this class of games, the difficulty of proving existence of usable and plausible equilibrium points, and the fact that closed-form solutions are possible only under very few specific functional forms.

We begin by describing the intended goals and limitations of this survey, the confines and special features of stochastic games in economics, and the general organization of this survey.

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<sup>1</sup> I would like to thank Jim Bergin, Manjira Datta, Igor Evstigneev, Wolfgang Leininger, Leonard Mirman, Jean-Francois Mertens, Abraham Neyman, Kevin Ref-fett, Matthew Sobel, Martin Shubik, and Sylvain Sorin for helpful exchanges concerning the subject of this paper.

<sup>2</sup> Shapley coined the term "stochastic games" by analogy to "stochastic processes", thus implicitly capturing the presence of dynamics. Since most applications actually involve models with deterministic transitions, this may appear somewhat misleading here, and "dynamic game" seems more appropriate. This is particularly true of studies considering open-loop equilibria, an essentially meaningless concept for games with chance moves.

### 14.1.1 Purpose and Scope of the Survey

This chapter provides a general survey of applications of stochastic games in economics and related fields. We identify clusters of studies according to methodological considerations (such as reliance on open-loop equilibrium, or perfect information, or computational simplicity), or to disciplinary categories (such as industrial organization or capital theory). The primary concern was to come up with convenient and natural categories that would be consistent with the general purpose of this volume while appealing to a diverse readership.

Coverage may be somewhat detailed and self-contained, or in the form of a brief summary with a listing of references, depending on how broadly used the particular framework under consideration has been, and on space considerations. In particular, for literature strands defined by a common methodological framework, a summary of the main results is provided. As the survey is organized both along methodological and subject lines, it is inevitable that some overlap will appear across different sections. Such occurrences are mentioned where appropriate so as to establish links between otherwise separate sections.

This survey will not encompass the continuous-time case, or differential games<sup>3</sup>, except in some cases where the results have direct qualitative analogs in discrete-time, or are otherwise of relevance to issues raised here. Likewise, although some links exist with the repeated games literature and with the standard two-stage game framework, these will not be dealt with here.

The survey is targeted more at potential users of the theory of dynamic games and young researchers in economics, rather than to game theorists or expert users. As a result, it seemed appropriate to review in some detail some important definitions, game-theoretic notions and frequently invoked facts from the theory of dynamic games. In particular, the presentation highlights the fact that these useful facts follow in a straightforward way from standard results in dynamic programming theory, which by now are familiar to most economists (see e.g. Stokey, Lucas and Prescott, 1989). On the other hand, for the sake of brevity, the reader is referred to other more self-contained or original sources for more detailed treatments and discussions, as well as for most proofs.

### 14.1.2 Special Features of Economic Applications of Dynamic Games

In relating the present survey to the rest of this volume, one must keep in mind that a number of motivations and widely held beliefs among economists have in large part shaped the nature and the focus of the studies invoking the theory of dynamic games in economics<sup>4</sup>. A brief account of these beliefs is now given.

<sup>3</sup> Basar and Olsder (1999) and Dockner et. al. (2000) are authoritative monographs on this related paradigm.

<sup>4</sup> Having said that, it is also true that different subfields of economics have been influenced by different disciplines (such as systems theory, or operations research, or mathematical game theory), and thus may reflect somewhat divergent practices and beliefs.