# 8. Isotone Recursive Methods: The Case of Homogeneous Agents

Manjira Datta and Kevin L. Reffett
Department of Economics, Arizona State University, Tempe, AZ, USA[1]

## 8.1 Introduction

A foundation of modern macroeconomics is the stochastic growth model originally introduced in the seminal work of Brock and Mirman[16]. Their original model is an infinite horizon economy with a continuum of identical households, each with access to a complete set of financial markets that insure them against all sources of idiosyncratic risk. There is single sector production that employs capital and labor whose returns are summarized by a stochastic neoclassical production function representing an aggregate convex production set with identical private and social returns to inputs. There is also aggregate risk taking the form of a collection of identically and independently distributed (i.i.d.) random variables, the agents in the economy face no frictions in information acquisition (i.e., there is no learning), labor supply is inelastic, and there are no equilibrium distortions. The authors characterize the unique *Markovian Equilibrium Decision Process* (MEDP) and its associated unique (non-trivial) long-run equilibrium dynamics, in particular, the *Stationary Markovian Equilibrium* (SME). Their methodological approach was pioneering, and relied heavily on recursive methods. Implicitly, it exploits the validity of a second welfare theorem and one can interpret the economic outcomes of the fictional social planner's problem

[1] We are deeply indebted to Len Mirman and Olivier Morand for numerous lengthy discussions concerning many issues discussed in this survey. Many of the results presented in this paper were developed originally in some form during our joint work with Len and Olivier over the last five years. We dedicate this paper to Len Mirman on the occasion of his sixty-fifth birthday. Indeed, this paper would not have been written without Len's ongoing pioneering work on equilibrium growth under uncertainty. We also thank Elena Antoniadou, Hector Chade, John Coleman, Jeremy Greenwood, Seppo Heikkilä, Ken Judd, Tom Krebs, Cuong Le Van, Robert Lucas, Jr., Jianjun Miao, Chris Shannon, John Stachurski, Yiannis Vailakis, Charles Van Marrewijk, Jean-Marie Viaene, Itzhak Zilcha, and especially Robert Becker and Manuel Santos for many helpful conversations over the past years. All mistakes remain our own.

from the perspective of a decentralized economic system. A fully decentralized recursive formulation of the Brock-Mirman framework is put forward by Prescott and Mehra[66] (see also, Stokey, Lucas, with Prescott[76]).

Over the last three decades, extensions of this model have become the foundation for the systematic study of many diverse issues in quantitative dynamic macroeconomic theory. Applications include models of economic fluctuations and business cycles, production-based asset pricing, the positive and normative implications of incomplete financial markets and public goods, the wealth inequality, the dynamic structure of altruistic economies, stochastic life-cycle models, models with physical and human capital, and the role of activist fiscal and/or monetary policy etc. However, many recent applications emphasize economic environments where the second welfare theorem is *not* available. These modifications create serious complications for a systematic study of the underlying structure of the MEDPs and the SME. A prevalent approach is to develop extensive applications of numerical methods to characterize MEDPs and the SME. From a mathematical perspective, many of these approaches have been *ad hoc* as they cannot be developed rigorously without providing characterizations of qualitative structure of the MEDPs and/or the SME.

An important question naturally emerges from this apparent disconnect between mathematical principle and macroeconomic practice: can one provide sharp and *constructive* characterizations of the MEDPs or the SME for generalized Brock-Mirman environments where the second welfare theorem fails? The most significant advance in providing an affirmative answer to this question has been the recent literature on "monotone methods" (also known as "monotone map" methods or "isotone recursive methods"). The pioneering work of Coleman [18][19][20][21], Greenwood and Huffman[34], Datta, Mirman, and Reffett[22] and Morand and Reffett[62] provide the genesis of the study of isotone recursive methods over the last fifteen years (they refer to them, as the "monotone-map" method). These papers present the first set of conditions under which constructive methods can be applied for studying the structure of a decentralized Markovian equilibrium in economies with or without non-classical production technologies.[2] An important generalization of this monotone-map approach is found in Mirman, Morand, and Reffett[59]. Here, a new and more general isotone map approach is presented (with the Coleman-Greenwood-Huffman approach as a special case) and can be applied to a larger collection of dynamic economies with production nonconvexities (in the reduced-form production function). In this setting, sets of sufficient conditions for the existence of semicontinuous, continuous, Lipschitz continuous, and

[2] The literature on monotone map methods is vast, and also includes the papers of Lucas and Stokey [55], Bizer and Judd [14] etc. An interesting alternative monotone method is developed in Becker and Foias [9].

For non-existence of a continuous MEDP, see Santos [73] and Krebs [50]. Mirman, Morand, and Reffett [59] show that although the Santos [73] example is robust to a large class of economies, in many case MEDPs are semi-continuous and isotone.