9. Discrete-Time Recursive Utility

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This chapter focuses on the fundamentals of discrete-time models using recursive utility. We examine the relation between preferences, utility, and aggregator, the existence of optimal paths, and several notions of impatience. In the one-sector model, we characterize optimal paths and derive a turnpike theorem.[1]

Topics beyond the scope of this paper include continuous time recursive utility, models involving uncertainty, the turnpike property in multisector models, and properties of Pareto optima and equilibrium in multisector models.[2]

Section 9.1 discusses the limitations of time additive preferences and some of the benefits of using a more general recursive utility specification. Section 9.2 examines the relation between recursive preferences and the associated aggregator function. A general result on existence of optimal paths is shown in Section 9.3. Sections 9.4 and 9.5 focus on the one-sector model. Existence of optimal paths and dynamic programming is considered in Section 9.4. Section 9.5 characterizes optimal paths via the Euler equations and then goes on to prove a one-sector turnpike theorem. Finally, Section 9.6 takes a brief look at the case where preferences are both homothetic and recursive.

9.1 Why Recursive Utility?

Since Ramsey [37], optimal growth models have primarily focused on the case of time additive separable (TAS) utility. Reasons for its popularity are easy to find. It is intuitively simple: We discount each period's utility at a constant

[1] For a more comprehensive treatment of the discrete-time case, see the book by Becker and Boyd [5].

[2] Epstein [19] examines when a recursive utility function is also a von Neumann-Morgenstern utility function. Existence and characterization of optimal paths is studied in [6] and [4]. Pareto optima and turnpikes in multisectoral models have been investigated by Epstein ([20], [21]) and Dana and Le Van ([13], [14], [15]).

rate before summing over time. It is often possible to obtain clear-cut analytic results. If a problem is not quite standard, a large amount of theory developed by mathematicians is readily applicable. It allows the use of dynamic programming.

In spite of these advantages, TAS utility also has some shortcomings. In particular, it builds in some assumptions about the marginal rate of substitution between consumption in different periods that may not be desirable. This is most obvious when considering consumption paths that are stationary. In that case, the marginal rate of substitution between consumption today and consumption in the following period is the inverse of the discount factor. It is unaffected by the level of consumption. If there are multiple consumption goods, this stationary marginal rate of substitution is also unaffected by the level of consumption of those other goods.[3]

This constant marginal rate of substitution severely constrains the long-run behavior of economic models. For example, a consumer facing a fixed interest rate will try either to save without limit, or to borrow without limit, except in the knife-edge case where the discount rate equals the interest rate.

This problem is especially severe when there are heterogeneous households. Unless all of the households have the same discount rate, the most patient household ends up with all the capital in the long run, while all other households consume nothing, using their labor income to service their debt (Becker, [3]). Recursive utility allows for upward (or downward!) sloping long-run capital supply curves and non-degenerate long-run wealth distributions.

The constant discount rate hypothesis also creates problems for the calculation of welfare losses arising from capital income taxation. In TAS models, the long-run supply of capital by households will be perfectly elastic at the discount rate. We are entitled to be a bit skeptical of the resulting welfare analysis.

When analyzing growing economies, the special behavior of TAS utility on paths that grow at a constant rate facilitates the construction of tractable models. Interestingly, there are non-TAS utility functions that exhibit the same behavior (Dolmas, [17]; Farmer and Lahiri[24]).

9.2 Recursive Utility and Aggregators

Alternative methods of aggregating a sequence of period utilities have long been proposed. Irving Fisher [25] suggested combining today's utility and tomorrow's utility as if they were two different consumption goods. The result could then be analyzed using indifference curves over present and future utility. Fisher's approach was formalized and axiomatized by Koopmans and his collaborators

[3] The use of TAS utility also requires that the intertemporal elasticity of substitution be equal to the coefficient of relative risk aversion. Epstein and Zin [23] used techniques borrowed from the recursive utility literature to construct Kreps-Porteus preferences that relax that restriction.