Notation and Convention

Here is a short list of current notation and convention used in the different chapters.

- We shall always assume that the underlying probability space \((\Omega, \mathcal{F}, P)\) is separable.
- Let \(\mathcal{A}(\subset \mathcal{F})\) be a \(\sigma\)-field, \(X\) an \(\mathcal{A}\)-measurable random variable and \(Y\) a random variable independent of \(\mathcal{A}\). Then, for any Borel function \(f\), the conditional expectation \(E[f(X,Y)|\mathcal{A}]\) shall be denoted as \(\hat{E}[f(X,Y)]\). In other words, the expectation concerns the hat-variables with all others remaining frozen.
- We sometimes make the abuse of notation: \(X \in \mathcal{A}\), meaning that the random variable \(X\) is \(\mathcal{A}\)-measurable.
- The symbol \(\gamma_t\) (resp. \(\delta_t\)) will only be used to denote the last (resp. the first) zero of a certain process, usually Brownian motion, before (resp. after) the time \(t\). We often abbreviate \(\gamma_1\) and \(\delta_1\) by \(\gamma\) and \(\delta\).
- In general, to a process \(N\), we associate \(\overline{N}\), its one sided supremum process; namely, for \(t \geq 0\), \(\overline{N}_t := \sup_{s \leq t} N_s\). However, for Brownian motion \((B_t; t \geq 0)\), we keep the usual notation \((S_t; t \geq 0)\) for its one-sided supremum.
- In this book, studies of the law of a process \((X_t; t \geq 0)\) often begin with: “For any bounded functional \(F\), \(E[F(X_s; s \leq t)]\ldots\). By this sentence, we mean that \(F\) is a measurable functional on \(C([0,t], \mathbb{R})\) if \(X\) is assumed to be continuous, on \(D([0,t], \mathbb{R})\) otherwise.
- \(\mathcal{F}^\sigma(X)\) will denote the initial enlargement of the filtration \((\mathcal{F}_t; t \geq 0)\) with the random variable \(X\), that is the filtration defined by

\[
\mathcal{F}_t^{\sigma(X)} := \bigcap_{\varepsilon > 0} (\mathcal{F}_{t+\varepsilon} \vee \sigma(X)), \quad t \geq 0
\]

We shall sometimes use the terminology: \(X\)-initial enlargement of \((\mathcal{F}_t; t \geq 0)\).
- For \(A : \Omega \rightarrow [0, \infty]\), a random time, we denote by \(\mathcal{F}^A\) the smallest filtration which contains \((\mathcal{F}_t; t \geq 0)\), and makes \(A\) a stopping time, i.e.
We shall sometimes use the terminology: $\Lambda$-progressive enlargement of $(\mathcal{F}_t; t \geq 0)$.

- All martingales considered in this volume are assumed to be càdlàg (i.e. right-continuous and left-limited); in a number of cases, they are even assumed to be continuous, but this will always be specified.
- $e$ (resp. $N$) will often denote a standard exponentially distributed variable (resp. a standard normal variable).
- The symbol $\hookrightarrow$ (resp. $\not\hookrightarrow$) denotes immersion (resp. non-immersion) between two filtrations $(\mathcal{F}_t; t \geq 0)$ and $(\mathcal{G}_t; t \geq 0)$ such that $\mathcal{F}_t \subseteq \mathcal{G}_t$ for every $t$; $(\mathcal{F}_t; t \geq 0)$ is said to be immersed in $(\mathcal{G}_t; t \geq 0)$ if all $(\mathcal{F}_t; t \geq 0)$-martingales are $(\mathcal{G}_t; t \geq 0)$-martingales. This notion will be studied in Chapter 5, but we already note that the more general situation when some (perhaps all...) $(\mathcal{F}_t; t \geq 0)$-martingales are $(\mathcal{G}_t; t \geq 0)$-semimartingales will be a recurrent subject of study in these lecture notes.

\[
\mathcal{F}^\Lambda_t := \bigcap_{\varepsilon > 0} (\mathcal{F}_{t+\varepsilon} \vee \sigma(A \wedge (t+\varepsilon))), \quad t \geq 0
\]