3 Dynamic Factor Models

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Summary. Factor models can cope with many variables without running into scarce degrees of freedom problems often faced in a regression-based analysis. In this article we review recent work on dynamic factor models that have become popular in macroeconomic policy analysis and forecasting. By means of an empirical application we demonstrate that these models turn out to be useful in investigating macroeconomic problems.

3.1 Introduction

In recent years, large-dimensional dynamic factor models have become popular in empirical macroeconomics. They are more advantageous than other methods in various respects. Factor models can cope with many variables without running into scarce degrees of freedom problems often faced in regression-based analyses. Researchers and policy makers nowadays have more data at a more disaggregated level at their disposal than ever before. Once collected, the data can be processed easily and rapidly owing to the now wide-spread use of high-capacity computers. Exploiting a lot of information can lead to more precise forecasts and macroeconomic analyses. The use of many variables further reflects a central bank's practice of 'looking at everything' as emphasized, for example, by Bernanke and Boivin (2003). A second advantage of factor models is that idiosyncratic movements which possibly include measurement error and local shocks can be eliminated. This yields a more reliable signal for policy makers and prevents them from reacting to idiosyncratic movements. In addition, the estimation of common factors or common shocks is of intrinsic interest in some applications. A third important advantage is that factor modellers can remain agnostic about the structure of the economy and do not need to rely on overly tight assumptions as is sometimes the case in structural models. It also represents an advantage over structural VAR models where the researcher has to take a stance on the variables to include which, in turn, determine

the outcome, and where the number of variables determine the number of shocks.

In this article we review recent work on dynamic factor models and illustrate the concepts with an empirical example. In Section 2 the traditional factor model is considered and the approximate factor model is outlined in Section 3. Different test procedures for determining the number of factors are discussed in Section 4. The dynamic factor model is considered in Section 5. Section 6 gives an overview of recent empirical work based on dynamic factor models and Section 7 presents the results of estimating a large-scale dynamic factor model for a large set of macroeconomic variables from European Monetary Union (EMU) member countries and Central and Eastern European Countries (CEECs). Finally, Section 8 concludes.

3.2 The Strict Factor Model

In an r-factor model each element of the vector $y_t = [y_{1t}, \ldots, y_{Nt}]'$ can be represented as

$$\begin{aligned} y_{it} &= \lambda_{i1} f_{1t} + \cdots + \lambda_{ir} f_{rt} + u_{it} \ , \quad t = 1, \ldots, T \\ &= \lambda'_{i\bullet} f_t + u_{it} \ , \end{aligned}$$

where $\lambda'_{i\bullet} = [\lambda_{i1}, \ldots, \lambda_{ir}]$ and $f_t = [f_{1t}, \ldots, f_{rt}]'$. The vector $u_t = [u_{1t}, \ldots, u_{Nt}]'$ comprises N idiosyncratic components and f_t is a vector of r common factors.

In matrix notation the model is written as

$$\begin{aligned} y_t &= \Lambda f_t + u_t \\ Y &= F\Lambda' + U, \end{aligned}$$

where $\Lambda = [\lambda_{1\bullet}, \ldots, \lambda_{N\bullet}]'$, $Y = [y_1, \ldots, y_T]'$, $F = [f_1, \ldots, f_T]'$ and $U = [u_1, \ldots, u_T]'$.

For the *strict factor model* it is assumed that u_t is a vector of mutually uncorrelated errors with $E(u_t) = 0$ and $E(u_t u_t') = \Sigma = \text{diag}(\sigma_1^2, \ldots, \sigma_N^2)$. For the vector of common factors we assume $E(f_t) = 0$ and $E(f_t f_t') = \Omega$.[1] Furthermore, $E(f_t u_t') = 0$. >From these assumptions it follows that[2]

$$\Psi = E(y_t y_t') = \Lambda \Omega \Lambda' + \Sigma.$$

The loading matrix Λ can be estimated by minimizing the residual sum of squares:

$$\sum_{t=1}^{T} (y_t - Bf_t)'(y_t - Bf_t) \tag{3.1}$$

[1] That is we assume that $E(y_t) = 0$. In practice, the means of the variables are subtracted to obtain a vector of mean zero variables.

[2] In many applications the correlation matrix is used instead of the covariance matrix of y_t. This standardization affects the properties of the principal component estimator, whereas the ML estimator is invariant with respect to a standardization of the variables.