4 Unit Root Testing*

Jürgen Wolters[1] and Uwe Hassler[2]

[1] Fachbereich Wirtschaftswissenschaft, Freie Universität Berlin
wolters@wiwiss.fu-berlin.de
[2] Fachbereich Wirtschaftswissenschaften, J.W. Goethe Universität Frankfurt
hassler@wiwi.uni-frankfurt.de

Summary. The occurrence of unit roots in economic time series has far reaching consequences for univariate as well as multivariate econometric modelling. Therefore, unit root tests are nowadays the starting point of most empirical time series studies. The oldest and most widely used test is due to Dickey and Fuller (1979). Reviewing this test and variants thereof we focus on the importance of modelling the deterministic component. In particular, we survey the growing literature on tests accounting for structural shifts. Finally, further applied aspects are addressed, for instance, how to get the size correct and obtain good power at the same time.

4.1 Introduction

A wide variety of economic time series is characterized by trending behaviour. This raises the important question how to statistically model the long-run component. In the literature, two different approaches have been used. The so-called trend stationary model assumes that the long-run component follows a time polynomial, which is often assumed to be linear, and added to an otherwise stationary autoregressive moving average (ARMA) process. The difference stationary model assumes that differencing is required to obtain stationarity, i.e. that the first difference of a time series follows a stationary and invertible ARMA process. This implies that the level of the time series has a unit root in its autoregressive (AR) part. Unit root processes are also called integrated of order 1, I(1).

Since the seminal paper by Nelson and Plosser (1982) economists know that modelling the long-run behaviour by trend or difference stationary models has far-reaching consequences for the economic interpretation. In a trend stationary model

*We thank Mu-Chun Wang for producing the figures, and an anonymous referee for comments improving the presentation.

the effects of shocks are only temporary implying that the level of the variable is not influenced in the long run. In contrast a shock has permanent effects in a difference stationary model, meaning that the level of the variable will be shifted permanently after the shock has occurred.

Traditional econometrics assumes stationary variables (constant means and time-independent autocorrelations). This is one of the reasons why applied economists very often transform non-stationary variables into stationary time series. According to the two above-mentioned models this can be done by eliminating deterministic trends in the case of a trend stationary model or by taking first differences in the case of a difference stationary model. But what happens if the wrong transformation is applied? The papers by Chan *et al.* (1977), Nelson and Kang (1981) and Durlauf and Phillips (1988) investigate this problem. Eliminating the non-stationarity in a trend stationary model by taking first differences has two effects: one gets rid of the linear trend, but the stationary stochastic part is overdifferenced, implying spurious short-run cycles. If, on the other hand, it is tried to eliminate the non-stationarity in a difference stationary model by taking the residuals of a regression on a constant and on time as explanatory variables, spurious long-run cycles are introduced. These depend on the number of observations used in the regression. In this case artificial business cycles are produced that lead to wrong economic interpretations.

Moreover, regressing independent difference stationary processes on each other leads to the problem of spurious regressions as Granger and Newbold (1974) have demonstrated in a simulation study. Later on Phillips (1986) gave the theoretical reasoning for this phenomenon: The usual t-statistics diverge to infinity in absolute value, while the R^2 does not converge to zero, hence indicating spurious correlation between independent difference stationary processes. Granger (1981) and Engle and Granger (1987) offered a solution to the spurious regression problem by introducing the concept of cointegration.

The above discussion clearly indicates that the analysis of non-stationary time series requires a serious investigation of the trending behaviour. Therefore, formal tests are needed which allow to distinguish between trend stationary and difference stationary behaviour of time series. Such tests have first been developed by Fuller (1976) and Dickey and Fuller (1979, 1981) (DF test, or augmented DF test, ADF). In the meantime a lot of extensions and generalizations have been published which also are presented in different surveys such as Dickey *et al.* (1986), Diebold and Nerlove (1990), Campbell and Perron (1991), Hassler (1994), Stock (1994) and Phillips and Xiao (1998).

Due to page limitations we will present here only the (augmented) Dickey-Fuller approach for testing the null hypothesis of difference stationarity. The related semi-parametric approach developed by Phillips (1987) and Phillips and Perron (1988) is not presented, and the extension to panel unit root tests is not considered, see Breitung and Pesaran (2005) for a recent overview. Furthermore, we do not deal with tests for seasonal unit roots as proposed e.g. by Hylleberg *et al.* (1990), or tests having stationarity in the maintained hypothesis as Kwiatkowski *et al.* (1992). We rather focus on modelling the deterministic part of the time series under investigation. This is very important in case of structural breaks, since neglecting