# 5 Autoregressive Distributed Lag Models and Cointegration*

Uwe Hassler[1] and Jürgen Wolters[2]

[1] Fachbereich Wirtschaftswissenschaften, J.W. Goethe Universität Frankfurt
hassler@wiwi.uni-frankfurt.de
[2] Fachbereich Wirtschaftswissenschaft, Freie Universität Berlin
wolters@wiwiss.fu-berlin.de

**Summary.** This paper considers cointegration analysis within an autoregressive distributed lag (ADL) framework. First, different reparameterizations and interpretations are reviewed. Then we show that the estimation of a cointegrating vector from an ADL specification is equivalent to that from an error-correction (EC) model. Therefore, asymptotic normality available in the ADL model under exogeneity carries over to the EC estimator. Next, we review cointegration tests based on EC regressions. Special attention is paid to the effect of linear time trends in case of regressions without detrending. Finally, the relevance of our asymptotic results in finite samples is investigated by means of computer experiments. In particular, it turns out that the conditional EC model is superior to the unconditional one.

## 5.1 Introduction

The autoregressive distributed lag model (ADL) is the major workhorse in dynamic single-equation regressions. One particularly attractive reparameterization is the error-correction model (EC). Its popularity in applied time series econometrics has even increased, since it turned out for nonstationary variables that cointegration is equivalent to an error-correction mechanism, see Granger's representation theorem in Engle and Granger (1987). By differencing and forming a linear combination of the nonstationary data, all variables are transformed equivalently into an EC model

*We thank Vladimir Kuzin for excellent research assistance and Surayyo Kabilova for skillful word processing. Moreover, we are grateful to an anonymous referee for clarifying comments.

with stationary series only.

Working on feedback control mechanisms for stabilization policy, Phillips (1954, 1957) introduced EC models to economics. Sargan (1964) used them to estimate structural equations with autocorrelated residuals, and Hendry popularized their use in econometrics in a series of papers[1]. According to Hylleberg and Mizon (1989, p. 124) '*the error correction formulation provides an excellent framework within which it is possible to apply both the data information and the information available from economic theory*'. A survey on specification, estimation and testing of EC models is given by Alogoskoufis and Smith (1995). The present paper contributes to this literature in that it treats some aspects of testing cointegration and asymptotic normal inference of the cointegrating vector estimated from an EC format.

The rest of the paper is organized as follows. The next section reviews different reparameterizations and interpretations of ADL models. Then we use that the cointegrating vector computed from the ADL model is equivalent to the one estimated from EC in order to use results by Pesaran and Shin (1998) on asymptotic normality. Section 4 turns to cointegration testing from EC regressions. We review t-type and F-type test statistics, and pay particular attention to the role of linear time trends. The relevance of our asymptotic results in finite samples is investigated through Monte Carlo experiments in Section 5. A detailled summary is contained in the final section.

## 5.2 Assumptions and Representations

The autoregressive distributed lag model of order $p$ and $n$, ADL($p$,$n$), is defined for a scalar variable $y_t$ as

$$y_t = \sum_{i=1}^{p} a_i \, y_{t-i} + \sum_{i=0}^{n} c_i' \, x_{t-i} + \varepsilon_t \, , \tag{5.1}$$

where $\varepsilon_t$ is a scalar zero mean error term and $x_t$ is a $K$-dimensional column vector process. Typically, a constant is included in (5.1), which we neglect here for brevity. The coefficients $a_i$ are scalars while $c_i'$ are row vectors. Using the lag operator $L$ applied to each component of a vector, $L^k \, x_t = x_{t-k}$, it is convenient to define the lag polynomial $a(L)$ and the vector polynomial $c(L)$,

$$\begin{aligned} a(L) &= 1 - a_1 L - \ldots - a_p \, L^p \, , \\ c(L) &= c_0 + c_1 L + \ldots + c_n \, L^n \, . \end{aligned}$$

Now, it is straightforward to write (5.1) more compactly:

$$a(L) y_t = c'(L) x_t + \varepsilon_t \, .$$

[1] Davidson *et al.* (1978), Hendry (1979), and Hendry *et al.* (1984). It is noteworthy that A.W. Phillips, Sargan as well as Hendry were professors at the London School of Economics. A personal view on the history of EC models is given in the interview of Hendry by Ericsson (2004).