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Message Sequence Charts

As discussed in detail in Chap. 5, ACATs are capable of modeling systems of several components that communicate via message exchange through fifo channels. Resuming that study, we now model those systems and their channels explicitly. Afterwards, an embedding of message-passing systems into ACATs allows the transfer of several results into this more specific setting. Remember that the behavior of those systems can be described by a collection of *message sequence charts* (MSCs). In the following, let us recall some basic definitions that have already been settled in Example 5.12. However, defining MSCs, we will now come from the more general notion of *partial* MSCs, which, in contrast to MSCs, admit events to be unmatched.

7.1 Message Sequence Charts

As usual, let Ag be a finite set of at least two agents. Towards defining a distributed alphabet that is tailored to channel systems, we first let

$$\begin{aligned} \Gamma_i^s(Ag) &:= \{i!j \mid j \in Ag \setminus \{i\}\} \\ \Gamma_i^r(Ag) &:= \{i?j \mid j \in Ag \setminus \{i\}\} \end{aligned}$$

denote the sets of *send* and, respectively, *receive actions* that are available to agent $i \in Ag$. Accordingly, $\Gamma_i(Ag) := \Gamma_i^s(Ag) \cup \Gamma_i^r(Ag)$ contains any action that i may execute, $\Gamma^s(Ag) := \bigcup_{i \in Ag} \Gamma_i^s(Ag)$ is the set of send, $\Gamma^r(Ag) := \bigcup_{i \in Ag} \Gamma_i^r(Ag)$ is the set of receive, and $\Gamma(Ag) := \Gamma^s(Ag) \cup \Gamma^r(Ag)$ is the set of all the actions. Here, the symbols $i!j$ and $j?i$ are to be read as "i sends a message to j" and "j receives a message from i", respectively. They are related in the sense that they will label communicating events of an MSC, which are joined by a message arrow in their graphical representation. We therefore set $Com(Ag) := \{(i!j, j?i) \mid (i,j) \in Ch(Ag)\}$. Recall that an action $i\theta j$ ($\theta \in \{!, ?\}$) is performed by agent i, which is indicated by $Ag(i\theta j) = i$.

In a natural manner, a message-passing system over Ag is based on the distributed alphabet $\widetilde{\Gamma}(Ag) := (\Gamma_i(Ag))_{i \in Ag}$. Henceforth, we mostly take the liberty of omitting the reference to Ag and just write Γ^s, Γ^r, Γ, *Com*, and $\widetilde{\Gamma}$, for example.

MSCs will be defined stepwise, starting with *partial MSCs*, where some events may lack a suitable communication partner. The latter view is concretized towards *lossy MSCs*, which allow at most unmatched send events, whereas, finally, a *basic MSC* describes complete behavior without any open events.

Definition 7.1 (Partial Message Sequence Chart). *A* partial message sequence chart *(over Ag) is a $\widetilde{\Gamma}$-dag $(V, \lhd, \lambda)$ such that*

- *for any $i \in Ag$, $\lhd_i = \lessdot_i$, i.e., $\lhd_i$ is the covering relation of $\leq_i$, and*
- *for any $(u, v) \in \lhd_{\mathrm{c}}$, $(\lambda(u), \lambda(v)) \in Com$.*

Unlike the general case of dags over distributed alphabets, the first condition from Definition 7.1 guarantees that the class of partial MSCs is locally covering. Moreover, the definition of a $\widetilde{\Gamma}$-dag makes sure that completed message transfers in a partial MSC, which correspond to an edge whose nodes are labeled with communicating actions, are processed along a fifo architecture. However, there might still be unmatched events. So suppose $\mathcal{M} = (V, \lhd, \lambda)$ is a partial MSC over Ag. We identify events from V that either have no communication partner yet or just lack a link to an existing partner and let V_{um} denote the set $\{u \in V \mid \text{there is no } v \in V \text{ such that } u \lhd_{\mathrm{c}} v \text{ or } v \lhd_{\mathrm{c}} u\}$ of *unmatched events* of $\mathcal{M}$. Given $u \in V$, $Ag(u)$ will subsequently serve as a shorthand for $Ag(\lambda(u))$. Moreover, we may write $Ag(\mathcal{M})$ instead of $\{Ag(u) \mid u \in V\}$.

Definition 7.2 (Lossy Message Sequence Chart). *A* lossy message sequence chart *is a partial MSC $(V, \lhd, \lambda)$ such that, for any $u \in V_{\mathrm{um}}$, $\lambda(u) \in \Gamma^s$.*

In other words, a lossy MSC has no unmatched receive events. In an MSC, finally, any event is part of a full message exchange:

Definition 7.3 (Message Sequence Chart). *A* message sequence chart *is a partial MSC $(V, \lhd, \lambda)$ such that we have $V_{\mathrm{um}} = \emptyset$.*

Example 7.4. An MSC $(V, \lhd, \lambda)$ over $\{1, 2, 3\}$ is depicted in Fig. 7.1b. However, to illustrate an MSC, we mostly represent it by a diagram such as shown in Fig. 7.1a, which is more intuitive and provides enough information to infer the corresponding graph. This example shows that it would be too restrictive if we confined ourselves to Hasse diagrams, i.e., to graphs from $\mathbb{DAG}_H(\widetilde{\Gamma})$, as the edge representing the second message from agent 1 to agent 2 is already implicitly present. Observe that we have $V_1 = \{u_1, u_2, u_3\}$, $u_1 \lhd_1 u_2 \lhd_1 u_3$, $u_2 \lhd_{\mathrm{c}} v_4$ and $u_2 \lhd_{(1,2)} v_4$, $w_3 \lhd_{\mathrm{c}} v_3$ and $w_3 \lhd_{(3,2)} v_3$, $v_1 \leq_2 v_4$, $u_3 \leq v_3$, $u_3 \lessdot w_2$, and $u_2 \not\lessdot v_4$.