9

Beyond Implementability

In this chapter, we turn our attention to the relation between MSO logic over MSCs and its existential fragment (and therefore, implicitly, CFMs, which refer to the notion of implementability). We also compare those logics to the classes of rational and recognizable MSC languages. In particular, MSO logic will turn out to be strictly more expressive than EMSO. Together with the results of the previous chapter, this will be used to prove that CFMs cannot be complemented in general and that they cannot be determinized. Those results rely on an encoding of grids into MSCs, which allows us to apply results from the framework of grids and graphs in general to MSCs. Altogether, we highlight the application limitations of CFMs so that future work might aim at finding large classes of CMs that still have promising algorithmic and logical properties.

9.1 EMSO vs. MSO in the Bounded Setting

Let us first study the corresponding problem in the bounded setting where we restrict the interpretation of formulas to $\forall B$-bounded MSCs.

Theorem 9.1. *For any $B \geq 1$,*

$$\mathcal{EMSO}_{\mathbb{MSC}_{\forall B}} = \mathcal{MSO}_{\mathbb{MSC}_{\forall B}} = \mathcal{EMSO}[\leq]_{\mathbb{MSC}_{\forall B}} = \mathcal{MSO}[\leq]_{\mathbb{MSC}_{\forall B}}$$

Proof. First note that the language of a CFM restricted to $\forall B$-bounded MSCs is regular. More precisely, for any CFM $\mathcal{A}$ and any $B \geq 1$, $L(\mathcal{A}) \cap \mathbb{MSC}_{\forall B}$ is a regular MSC language. With Theorem 8.33 and results from [45] and [56], this implies $\mathcal{EMSO}_{\mathbb{MSC}_{\forall B}} = \mathcal{EMSO}[\leq]_{\mathbb{MSC}_{\forall B}} = \mathcal{MSO}[\leq]_{\mathbb{MSC}_{\forall B}}$.

It remains to show that $\mathcal{EMSO}_{\mathbb{MSC}_{\forall B}} \supseteq \mathcal{MSO}_{\mathbb{MSC}_{\forall B}}$. More generally, we show that, for each $\varphi(Y_1, \ldots, Y_n) \in$ MSO, $n \geq 1$, there is a CFM $\mathcal{A}$ (adapted to structures from $\langle \mathbb{MSC}, \{0,1\}^n \rangle$) such that $L(\mathcal{A}) = L_{\mathbb{MSC}_{\forall B}}(\varphi)$. So let $\varphi(Y_1, \ldots, Y_n) \in$ MSO, which, according to Lemma 3.5, can be assumed to be of the form

$$\exists \overline{X_k} \forall \overline{X_{k-1}} \ldots \exists/\forall \overline{X_1} \psi(Y_1, \ldots, Y_n, \overline{X_k}, \ldots, \overline{X_1})$$

or, equivalently,

$$\exists \overline{X_k} \neg \exists \overline{X_{k-1}} \ldots \neg \exists \overline{X_1} \psi'(Y_1, \ldots, Y_n, \overline{X_k}, \ldots, \overline{X_1})$$

for some $k \geq 1$. We proceed by induction on k. For $k = 1$, φ is an EMSO-formula, which has an equivalent CFM counterpart $\mathcal{A}$ (tailored to extended MSCs), i.e., $L(\mathcal{A}) = L_{\mathbb{MSC}}(\varphi)$. Using $\Longrightarrow_{\mathcal{A}}$, we gain some finite automaton over $\Gamma \times \{0,1\}^n$ recognizing $Lin(L(\mathcal{A}) \cap \langle \mathbb{MSC}_{\forall B}, \{0,1\}^n \rangle)$, which is a witness for the fact that $L(\mathcal{A}) \cap \langle \mathbb{MSC}_{\forall B}, \{0,1\}^n \rangle$ is a regular MSC language. According to [74], there is $\mathcal{A}' \in$ det-$\forall$CFM with $L(\mathcal{A}') = L(\mathcal{A}) \cap \langle \mathbb{MSC}_{\forall B}, \{0,1\}^n \rangle$. (Though we did not explicitly define what determinism means for extended CMs, it is obvious how to adjust the definition accordingly so that, then, the above-mentioned result by Mukund et al. also holds in the extended setting.) Induction now alternately involves complementation and projection steps. A complementation step first requires the construction of the CFM $\overline{\mathcal{A}'}$ from $\mathcal{A}'$ with $L(\overline{\mathcal{A}'}) = \langle \mathbb{MSC}, \{0,1\}^n \rangle \setminus L(\mathcal{A}')$, which, though taking into account that we deal with extended MSCs, can be found along the usual lines: we first provide a complete deterministic CFM, whose set of global final states is then complemented. Projection is even easier, as communication actions just need to be projected onto the remaining components. □

As sets of $\forall B$-bounded MSCs can be seen as sets of Mazurkiewicz traces [56] and, in the setting of Mazurkiewicz traces, MSO logic is expressively equivalent to asynchronous automata (cf. Theorem 6.22), Theorem 8.33 can be understood as an extension of Zielonka's theorem.

Proposition 9.2 ([56]). *For any $B \geq 1$, the following hold:*

(a) $\mathbb{MSC}_{\forall B} \in \mathcal{EMSO}_{\mathbb{MSC}}$,
(b) $\mathbb{MSC}_{\forall B} \in \mathcal{EMSO}[\leq]_{\mathbb{MSC}}$.

By Proposition 9.2, Theorem 9.1 can be sharpened as follows:

Theorem 9.3. *For any $\forall$-bounded MSC language L, the following statements are equivalent:*

1. $L \in \mathcal{EMSO}_{\mathbb{MSC}}$,
2. $L \in \mathcal{MSO}_{\mathbb{MSC}}$,
3. $L \in \mathcal{EMSO}[\leq]_{\mathbb{MSC}}$,
4. $L \in \mathcal{MSO}[\leq]_{\mathbb{MSC}}$,
5. $L \in \mathcal{CFM}$.

It was even shown that, if we restrict to $\exists$-bounded MSC languages, any $\mathrm{MSO}_{\mathbb{MSC}}$-definable set is implementable.

Theorem 9.4 ([35]). *Theorem 9.3 holds for $\exists$-bounded MSC languages verbatim.*